Engineering Mathematics -IV (MA2217)_Assignments_1 &_2

Q1. A and B play a game in which their chances of winning are in the ratio 3: 2. Find A's chance of winning at least three games out of the five games played.

Q2. The chance that Doctor A will diagnose a disease X correctly is 60%. The chance that the patient will die by his treatment after correct diagnosis is 40% and the chance of death of wrong diagnosis is 70%. A Patient of Doctor A, who had disease x died. What is the chance that this disease was diagnosed correctly?

Q3. A computer center has three printers A, B, and C, which print at different speeds. Programs are routed to the first available printer. The probability that the programs are routed to the printers A, B and C are 0.6, 0.3 and 0.1 respectively. Occasionally, a printer will jam and destroy a Printout. The probability that printers A, B and C will jam are 0.01, 0.05 and 0.04 respectively. Your program is destroyed when a printer jams. What is the probability that printer A is involved?

Q3. From a lot of 10 items containing 3 defective items, a sample of 4 items is drawn at random. Let the random variable X denote the number of defective items in the sample. Answer the following when the sample is drawn without replacement.

(i) Find the probability distribution of X. (ii) Find P(X≤1), P(X<1) and P(0<X<1).

Q4. The diameter of an electric cable, say X is assumed to be a continuous random variable with probability density function given by f(x) = kx (1 – x), 0 ≤ x ≤1. Determine k and P(X≤ 1/3).

Q5. The probability function of an infinite discrete distribution is given by

$P(X = j) = 1/2^j (j = 1,2,3 \ldots .)$. Verify that the total probability is one and find the mean and variance of the distribution. Find also P (X is even), P(X≤ 5) and P(X is divisible by 3).

Q6. A continuous random variable X that can assume any value between X = 2 and X = 5 has a density function given by $f(x) = k(1 + x)$. Find P(X< 4).

Q7. The joint probability density function of a two dimensional random variables (X,Y) is given by $f(x, y) = xy^2 + \frac{x^2}{8}, 0 \leq x \leq 2, 0 \leq y \leq 1$. Compute $P(X > 1), P(Y < 1/2)$, $P(X > 1/Y < 1/2)$.

Q8. A gun is aimed at a certain point (origin of the coordinate system). Because of the random factors, the actual hit point can be any point (X, Y) in a circle of radius R about the origin. Assume that the joint density of X and Y is constant in the circle given by $f_{x,y}(x, y) = C, \ for \quad x^2 + y^2 \leq R^2$

= 0, otherwise.

Compute C.

Q9. The joint probability mass function of (X, Y) is given by $p(x, y) = k(2x + 3y), x =$ 0,1,2; y=1,2,3. Find (i) $P(X = i/Y = 1)$ (ii) probability distribution of (X+Y).

Q10. The current I and the resistance R in a circuit are independent continuous random variables with the following density functions.

$$f_i(i) = 2i, \;\; 0 \leq i \leq 1$$
$$= 0, \quad \text{elsewhere}$$
$$f_r(r) = r^2/9, \;\; 0 \leq r \leq 3$$
$$= 0, \quad \text{elsewhere}$$

Find the probability density function of the voltage E in the circuit where E= IR.

Q11. If the random variable X takes the values 1,2,3 and 4 such that $2P(X = 1) = 3P(X = 2) = P(X = 3) = 5P(X = 4)$,find the probability distribution function and cumulative distribution function of X.

Q12. If 20% of the bolts produced by a machine are defective, determine the probability that, out of 4 bolts chosen at random. (i) 1, (ii) 0, and (iii) at most 2 bolts will be defective.

Q13. The probability that an entering university student will graduate is 0.4. Determine the probability that out of 5 students (i) none, (ii) at least 1, and (iii) all will graduate.

Q14. What is the probability of getting a total of 9 (i) twice and (ii) at least twice in 6 tosses of a pair of dice?

Q15. An insurance company insures 4000 people against loss of both eyes in a car accident. Based on previous data, the rates were computed on the assumption that on the average 10 persons in I,00,000 will have car accident each year that results in this type of injury. What is the probability that more than 3 of the insured will collect on their policy in a given year?

Q16. Car hire firm has 2 cars, which it hires out day by day. The number of demands for a car on each day is distributed as a Poisson distribution with mean 1.5. Calculate the proportion of days on which (i) car is used, and (ii) the proportion of days on which some demand is refused.

Q17. X normally distributed and the mean of X is 12 and extended b series 4. find out the probability of the following:

(i) X ≥ 20 (ii) X≤ 20 and (iii) 0 ≤ X ≤ 12.

Q18. The marks obtained by a number of students for a certain subject are assumed to be approximately normally distributed with mean value 65 and with a standard deviation of 5. If 3 students are taken at random from this set, what is the probability that exactly 2 of them will have marks over 70?

Q19. In a distribution exactly normal, 10.03% of the items are under 25-kilogram weight and 89.97% of the items are under 70-kilogram weight. What are the mean and standard deviation of the distribution?

Q20. In an examination it is laid down that a student passes if he secures 30% or more marks. He is placed in the first, second or 3rd division according as he secures 60% or more marks, between 45% and 60% Marks and marks between 30% and 45% respectively. He gets distinctions in case he secures 80% or more marks. It is not noticed from the result that 10% of the students failed in the examination, whereas 5% of them of trained distinction. Calculate the percentage of the students placed in the second division. (Assume normal distribution of marks.)

Q21. Subway trains on a certain line run every half hour between midnight and 6:00 in the morning. What is the probability that a man entering the station at a random time during this period will have to wait at least 20 minutes?

Q22. If X is uniformly distributed with mean and variance 4/3. Find P (X < 0).

Q23. Marks obtained by a number of students are found to be distributed normally with mean 64 and variance 25. If 3 students are taken randomly find the probability that 2 of them have marks more than 70.

Q24. The time (in hours) required to repair a machine is exponentially distributed with parameter β=1/2. (i) What is the probability that repair time exceeds 2 hours. (ii) What is the conditional probability that a repair takes at least 10 hours given that its duration exceeds 9 hours?

Q25. If X has an exponential distribution with mean = 2, find $P\{(X < 1)/X < 2)\}$.

Q26. In an intelligence test administered on 1000 students, the average was 42 and standard deviation 24. Find

(i) the number of students exceeding a score 50,
(ii) the number of students lying between 30 and 54.
{Given $\emptyset(z = 0.333) = 0.3696, \emptyset(z = 0.5) = 0.1915$}

Q27. There are 3 true coins and one false coin with 'head' on both sides. A coin is chosen at random and tossed 4 times. If 'head' occurs all the 4 times what is the probability that the false coin has been chosen and used?

Q28. A bolt is manufactured by 3 machines A, B and C. A turns out twice as many items as B, and machines B and C produce equal number of items. 2% of the bolts produced by A and B are defective and 4% of bolts produced by capital C are defective. All bolts are put into 1 stock pile and 1 is chosen from this pile. What is the probability that it is defective?

Q29. For a certain binary communication channel the probability that a transmitted '0' is received as a '0' is 0.95 and the probability that a transmitted '1' is received as '1' is 0.90. If the probability that '0' is transmitted is 0.4. Find the probability that (i) a '1' is received and (ii) a '1' was transmitted given that '1' was received.

Q30. Starting at 5:00 AM every half hour there is a flight from San Francisco airport to Los Angeles International Airport. Suppose that none of these planes is completely sold out and that they always have room for passengers. A person who wants to fly to Los Angeles arrives at the airport random time between 8:45 AM and 9:45 AM. Find the probability that She waits (i) at most 10 minutes, and (ii) at last 15 minutes.

Q31. The mileage which car owners get with certain kind of radial tyre is a random variable having an exponential distribution with mean 40000 kilometre. Find the probabilities that one of these tyres will last (i) at least 20,000 kilometres and (ii) at most 30,000 kilometres.

Q32. A company has two plants to manufacture scooters plant I manufacturers 80% of the scooters and plant II the rest. At plant I, 85 out of 100 scooters are rated higher quality and at plant II, only 65 out of the 100 scooters are rated higher quality. A scooter is chosen at random. What is the probability that the scooter came from plant II, if it is known that the scooter is of higher quality?