Department of Electrical Engineering,
College of Engineering, University of North Texas, Denton
Control Systems Design (EENG5310-001)
Take Home Midterm Examination-3 (Fall 2023)

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Learner:

Time: 1 Week **Total Points: 45**

Q.1. (a) Present the state space representation in **controllable** canonical forms for a control system with the following transfer function. In other words, develop equations of the form $\dot{\boldsymbol{X}} = \boldsymbol{A}.\boldsymbol{X} + \boldsymbol{B}.\boldsymbol{u}$ and $\boldsymbol{Y} = \boldsymbol{C}.\boldsymbol{X} + \boldsymbol{D}.\boldsymbol{u}$, where ***A***, ***B***, ***C***, and ***D*** are the so called time-invariant *system, input, output*, and *feedforward* matrices, respectively, and ***X***, ***u***, and ***Y*** are system state, control, and output column vectors (or scalars) that are variables of time:

$$\frac{Y(s)}{U(s)} = \frac{s^2 + 2s + 3}{s^3 + 5s^2 + 8s + 4}$$ **(2 Points)**

(b) For a problem with an ***A*** (in controllable canonical form) that has one or more eigen values of multiplicity greater than 1, a matrix $S = \begin{bmatrix} 1 & & \\ \lambda_1 & & \\ \lambda_1^2 & & \end{bmatrix}$ with second column elements as derivatives with respect to λ_1 of the corresponding first column elements, in case λ_1 repeats twice, and third column elements as derivatives of the second column elements divided by 2, in case multiplicity of λ_1 is 3, and so on, would convert the matrix A into a Jordan matrix J using the formula $S^{-1} A\, S = J$. Show that the state matrix ***A*** of the above system has an eigen value with multiplicity > 1, and hence can be transformed into matrix ***J*** with a Jordan sub-matrix block using the state variable transformation $\boldsymbol{S}^{-1}\boldsymbol{A}\,\boldsymbol{S} = \boldsymbol{J}$. Indicate the general form of the matrix ***S***, and obtain ***J*** for the system of **1. (a)** using ***S*** specific to this problem.

(4 Points)

(c) Show that the state transition matrix $\boldsymbol{\emptyset(t)}$ for this problem is given by the formula $\boldsymbol{\emptyset(t)} = \boldsymbol{e^{At}} = \boldsymbol{S e^{Jt} S^{-1}}$ and hence find $\boldsymbol{\emptyset(t)}$ for the problem in 1(a). **(4 Points)**

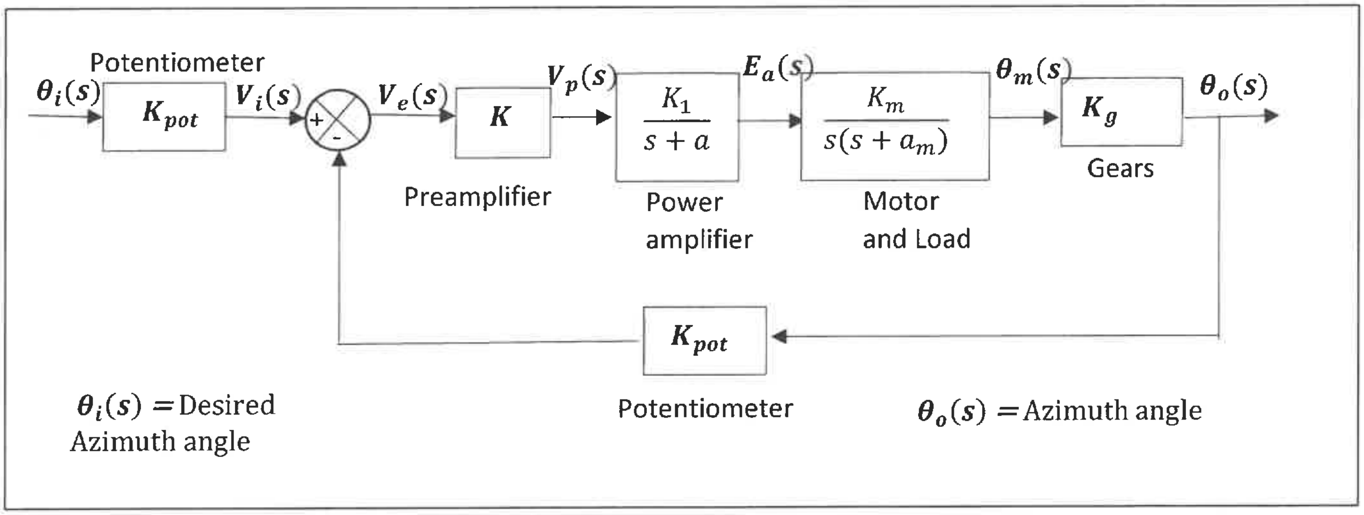
(d) Assuming zero initial conditions for the state variables, show that the system output $\boldsymbol{Y(t)}$ is given by $\boldsymbol{Y(t) = C.X(t) + D.u(t) = C.\int_0^t \emptyset(t-\tau)\, B\ u(\tau) d\tau + D.u(t)}$, and then obtain the unit step response of the system. **(5 Points)**

(e) Show that $\emptyset(t) = e^{At} = \mathcal{L}^{-1}[(sI - A)^{-1}]$ and $Y(t)$ is given by $Y(t) = \mathcal{L}^{-1}[\{C(sI - A)^{-1}B + D\}U(s)]$ when initial state vector $\boldsymbol{X(0) = 0}$. Here $\mathcal{L}^{-1}$ indicates inverse Laplace transform. Now, using this formula, find the unit step response of the system in 1 (a) and tally the result with that in **1(d).**

(5 Points)

2. Since there is not enough time for doing a project and writing a detailed report, I am using the following design problem as a substitute for the project. **(25 Points)**

Design Problem: The layout and schematic for an antenna azimuth control system are given separately on page 3. The block diagram for the system is given below:



The parameters of the system are: $K_{pot} = 0.318, K = 200, K_1 = 100, a = 100, K_m = 2.083, a_m = 1.71, and\ K_g = 0.1$. Since input gain and the feedback gain are the same $\boldsymbol{K_{pot}}$, we may replace the two $\boldsymbol{K_{pot}}$ blocks by a single one after the summation element thereby making it a unity feedback system. Finally, *we remove the unity feedback and make it an open-loop system* that would be modified to yield desired performance with the help of state feedback controller.

For this system, design a state feedback controller so that the overall system meets the requirements of peak overshoot of 10% and 5% setting time of 1 second. Assume that the real pole of the third order system is ten times as far from the imaginary axis as the dominant complex conjugate pair of the underlying second order system. You need to show first that the system is controllable because arbitrary pole placement is only possible only if the system is controllable.

Since the state variables of the system are inaccessible, you have to determine first whether the system is observable, and then design an observer (a replica of the original system) ten times as fast as the original system with state feedback and apply state feedback to the original system using estimated state variables.

(Note that observer can be made ten times as fast by placing the observer's real pole ten times farther away from that of the target system, and its natural frequency 10 times that of the dominant 2nd order system underlying the target system).

Draw the plant with the original system with state feedback provided from the observer.

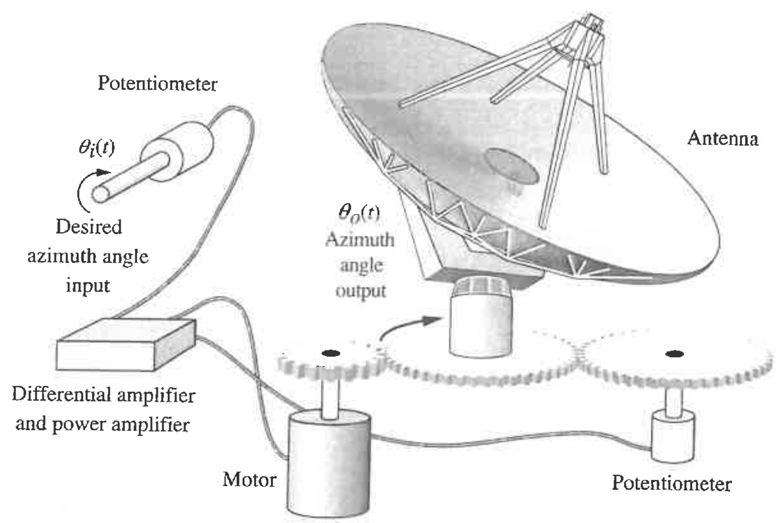
Draw plots to compare the impulse responses of the actual system and the observer for two situations:

i) the initial states of state variables both the original system and observer are zero.
ii) they are 0.006 for the original plant and 0 for the estimated state variables.

(Antenna Layout and schematic on the next page)

Antenna Azimuth Position Control System

Layout



Schematic