Kingston University London

School of Engineering and the Environment

Department of Mechanical Engineering

Coursework Assessment Brief

Module Code	EG4017
Module Title	Engineering Mathematics
Title of Assessment	Engineering Mathematics and Computing: Task 1
Summative (% of module)	Summative - this assignment is worth 50% of your module grade.
Typical individual student hours required to complete the assessment	Total hours per student is 25 hours
Assessment set by (and contact)	Dr Sergey Khaustov, RVMB122 *S.Khaustov@kingston.ac.uk*
Submission deadline (date and time)	7th November 2023, *before* 5pm
Formal feedback	Feedback will be released by 20 working days after the submission.

All assignments must be submitted by the date and time specified above.
Students are required to submit an electronic copy of their completed assignment via the Assignments section of Canvas and follow any specific instructions. Any change to this instruction will be advised via Canvas.

In line with Faculty policy for late submission of coursework, any work submitted up to a week late will be capped at 40%. Coursework submitted after this time will receive 0%.

In case of illness or other issues affecting your studies please refer to the University Mitigating Circumstances policy. Guidance on mitigating circumstances can be found on MyKingston:

https://mykingston.kingston.ac.uk/myfaculty/sec/secstudentsupportMC/Pages/Mitigating-Circumstances.aspx

Please note that if you submit a piece of work you have judged yourself fit to undertake the assessment and cannot claim mitigating circumstances retrospectively.

Guidance on avoiding academic assessment offences such as plagiarism and collusion can be found on MyKingston

https://mykingston/myuni/academicregulations/Pages/default.aspx

Module Learning Outcomes

The following module learning outcomes and professional body learning outcomes are tested in this assessment:

1) Display fluency in algebraic and numerical manipulations of functions (such as polynomial, rational, trigonometric, exponential, and logarithmic) and in manipulating complex numbers including finding powers and roots of complex numbers
2) Formulate mathematical solutions involving differentiation, integration, matrices, vector analysis, geometry and trigonometry and evaluate statistical data and probability through suitable software such as MATLAB

Assessment task and specific terms

Part A: Mathematics [20 marks]

1. Use the quotient rule to differentiate the function $y = \frac{\ln 3x}{2x}$

   [2 marks]

2. Find the angle between the vectors $2i - 11j - 10k$ and $5i + 8j + 7k$

   [2 marks]

3. Find the rate of change of $y = \ln(16t^2 + 19)$ at the specified point $t = 9$

   [2 marks]

4. Express $\cos t - 8\sin t$ in the form $A\cos(\omega t + \alpha)$, where $\alpha \geq 0$

   [2 marks]

5. Solve the following system of three linear equations using Cramer's rule

$$\begin{cases} 11v_1 - v_2 + v_3 = 31.4 \\ v_1 + \frac{v_2}{2} - v_3 = 1.9 \\ -9v_1 + 11v_3 = -12 \end{cases}$$

   [2 marks]

6. Transpose $z = d + a\sqrt{y}$ to make y the subject

   [2 marks]

7. Find a vector that is perpendicular to both of the vectors $\boldsymbol{a} = 4\boldsymbol{i} + 3\boldsymbol{j} + 5\boldsymbol{k}$ and $\boldsymbol{b} = 3\boldsymbol{i} + 4\boldsymbol{j} - 6\boldsymbol{k}$. Hence find a unit vector that is perpendicular to both $\boldsymbol{a}$ and $\boldsymbol{b}$.

   [2 marks]

8. If $M = \begin{pmatrix} 7 & 9 \\ 1 & -2 \end{pmatrix}$ and $N = \begin{pmatrix} 2 & 1 \\ -2 & 6 \end{pmatrix}$ find MN and NM

   [2 marks]

9. If $y = x^4 - 4x^3 - 90x^2$, find the values of x for which $y'' = 0$

   [2 marks]

10. Transpose $b = g + t(a - 3)$ to make a the subject

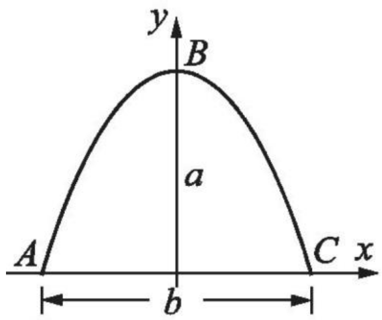
   [2 marks]

Part B: Computing (MATLAB) [30 marks]

Question 1

The arc length of a segment of a parabola ABC of an ellipse with semi-minor axes a and b is given approximately by:

$$L_{ABC} = \frac{1}{2}\sqrt{b^2 + 16a^2} + \frac{b^2}{8a}\ln\left(\frac{4a + \sqrt{b^2 + 16a^2}}{b}\right)$$



Write a universal, user-friendly code, test your programme and determine L_{ABC} if a = 11 cm and b = 9 cm.

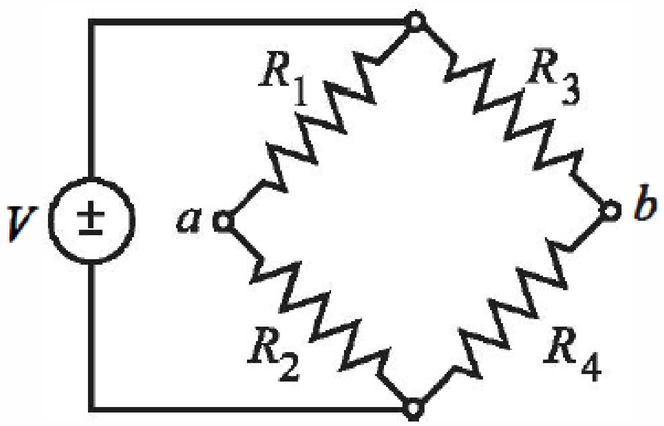
[1 mark]

Question 2

The voltage difference V_{ab} between points a and b in the Wheatstone bridge circuit is:

$$V_{ab} = V\left(\frac{R_1R_3 - R_2R_4}{(R_1 + R_2)(R_3 + R_4)}\right)$$

Write a universal, user-friendly code. Test you programme calculating the voltage difference when V = 14 volts, R_1 = 120.6 ohms, R_2 = 119.3 ohms, R_3 = 121.2 ohms, and R_4 = 118.8 ohms.



[1 mark]

Question 3

Newton's law of cooling gives the temperature $T(t)$ of an object at time tin terms of T_0, its temperature at $t = 0$, and T_s, the temperature of the surroundings.

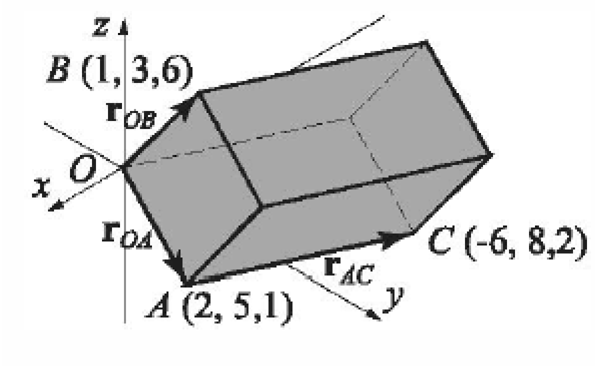
$$T(t) = T_s + (T_0 - T_s)e^{-kt}$$

A police officer arrives at a crime scene in a hotel room at 9:18PM, where he finds a dead body. He immediately measures the body's temperature and find it to be 26.4° C. Exactly one hour later he measures the temperature again, and find it to be 25.5° C. Determine the time of death, assuming that victim body temperature was normal (36.6° C) prior to death, and that the room temperature was constant at 20.5° C.

[3 marks]

Question 4

The volume of the parallelepiped shown can be calculated by $r_{OB} \cdot (r_{OA} \times r_{AC})$. Use the following steps in a script file to calculate the area. Define the vectors r_{OA}, r_{OB}, and r_{AC} from inputted positions of points A, B, and C. Determine the volume by using MATLAB 's built-in functions *dot* and *cross*.



[2 marks]

Question 5

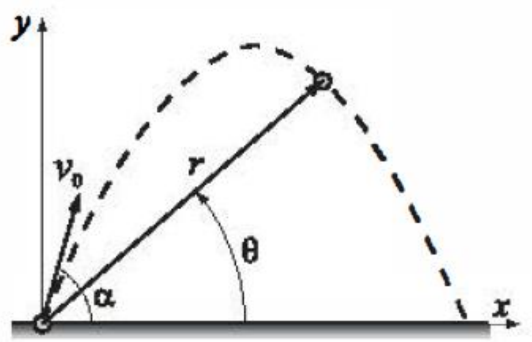
The position as a function of time ($x(t)$,$y(t)$) of a projectile fired with a speed of v_0 at an angle α is given by

$$x(t) = v_0 \cos\alpha \cdot t,\, y(t) = v_0 \sin\alpha \cdot t - \frac{1}{2}gt^2$$

where $g = 9.81$ m/sec^2. The polar coordinates of the projectile at time t are ($r(t)$, $\theta(t)$), where

$$r(t) = \sqrt{x^2(t) + y^2(t)},\ \tan\theta(t) = \frac{y(t)}{x(t)}$$

Write a universal, user-friendly code. Test case: $v_0 = 162$ m/sec and $\alpha = 70°$. Determine $r(t)$ and $\theta(t)$ for $t = 1, 6, \ldots, 31$ sec.



[3 marks]

Question 6

The ideal gas equation states that $P = \frac{nRT}{V}$, where P is the pressure, V is the volume, T is the temperature, R = 0.08206 (L atm)/(mol K) is the gas constant, and n is the number of moles. Real gases, especially at high pressure, deviate from this behaviour. Their response can be modelled with the van der Waals equation

$$P = \frac{nRT}{V - nb} - \frac{n^2 a}{V^2}$$

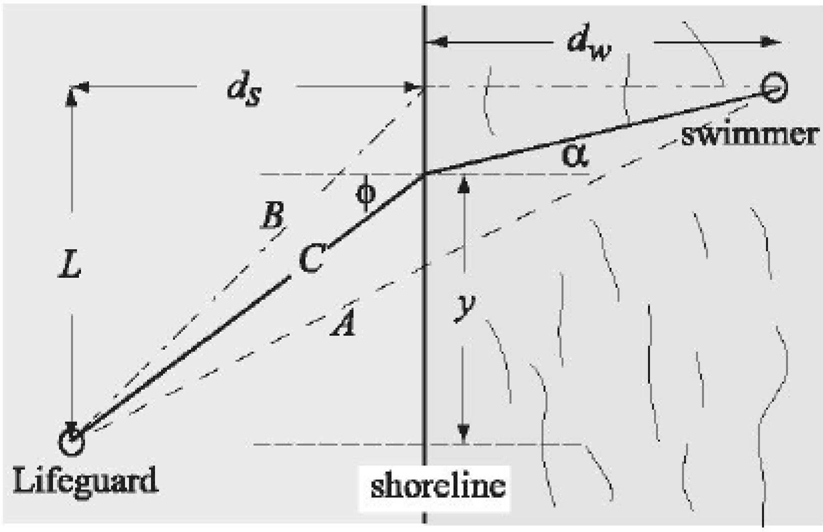
where a and b are material constants. Consider 1 mole (n = 1) of nitrogen gas at T = 300K. (For nitrogen gas a = 1.39 (L^2 atm)/mol^2, and b = 0.0391 L/mol.) Create a vector with values of Vs for $0.1 \leq V \leq 1$ L, using increments of 0.02 L. Using this vector calculate P twice for each value of V, once using the ideal gas equation and once with the van der Waals equation. Using the two sets of values for P, calculate the percent of error $\left(\frac{P_{ideal} - P_{waals}}{P_{waals}}\right)$ for each value of V. Finally, by using MATLAB's built-in function *max*, determine the maximum error and the corresponding volume.

[3 marks]

Question 7

A student has a summer job as a lifeguard at the beach. After spotting a swimmer in trouble, they try to deduce the path by which they can reach the swimmer in the shortest time. The path of shortest distance (path A) is obviously not the best since it maximizes the time spent swimming (they can run faster than they can swim). Path B minimizes the time spent swimming, but is probably not the best, since it is the longest (reasonable) path. Clearly the optimal path is somewhere in between paths A and B.
Consider an intermediate path C and determine the time required to reach the swimmer in terms of the running speed $v_{run} = 3$ m/sec, the swimming speed $v_{swim} = 1$ m/sec, the distances $L = 48$ m, $d_s = 30$ m, and $d_w = 42$ m; and the lateral distance y at which the lifeguard enters the water. Create a vector y that ranges between path A and path B (y = 20, 21, 22, ... , 48 m) and compute a time t for each y. Use MATLAB built-in function *min* to find the minimum time, and the entry pointy for which it occurs. Determine the angles that correspond to the calculated value of y and investigate whether your result satisfies Snell's law of refraction:

$$\frac{\sin\phi}{\sin\alpha} = \frac{v_{run}}{v_{swim}}$$



Is there any other way to optimise the path?

[3 marks]

Question 8

In a typical tension test a dog bone shaped specimen is pulled in a machine. During the test, the force needed to pull the specimen and the length L of a gauge section are measured. This data is used for plotting a stress-strain diagram of the material. Two definitions, engineering and true, exist for stress and strain. The engineering stress σ_c and strain ε_c are defined by $\sigma_c = \frac{F}{A_0}$ and $\varepsilon_c = \frac{L-L_0}{L_0}$, where L_0 and A_0 are the initial gauge length and the initial cross-sectional area of the specimen, respectively. The true stress σ_t and strain ε_t are defined by $\sigma_t = \frac{FL}{A_0 L_0}$ and $\varepsilon_t = \ln\frac{L}{L_0}$.
The following are measurements of force and gauge length from a tension test with an aluminium specimen. The specimen has a round cross section with radius 6.4 mm (before the test). The initial gauge length is L_0 = 25 mm. Use the data to calculate and generate the engineering and true stress-strain curves, both on the same plot. Label the axes and use a legend to identify the curves. Units: When the force is measured in newtons (N) and the area is calculated in m^2, the unit of the stress is pascals (Pa).

F, N	0	13.031	21.485	31.3963	34.727	37.119	37.960	39.550	40.758
L, mm	25.400	25.474	25.515	25.575	25.615	25.693	25.752	25.978	26.419
F, N	40.986	41.076	41.255	41.481	41.564				
L, mm	26.502	26.600	26.728	27.130	27.441				

[3 marks]

Question 9

A railroad bumper is designed to slow down a rapidly moving railroad car. After a 20,000 kg railroad car traveling at 20 m/sec engages the bumper, its displacement x (in meters) and velocity v (in m/sec) as a function of time t (in seconds) is given by:

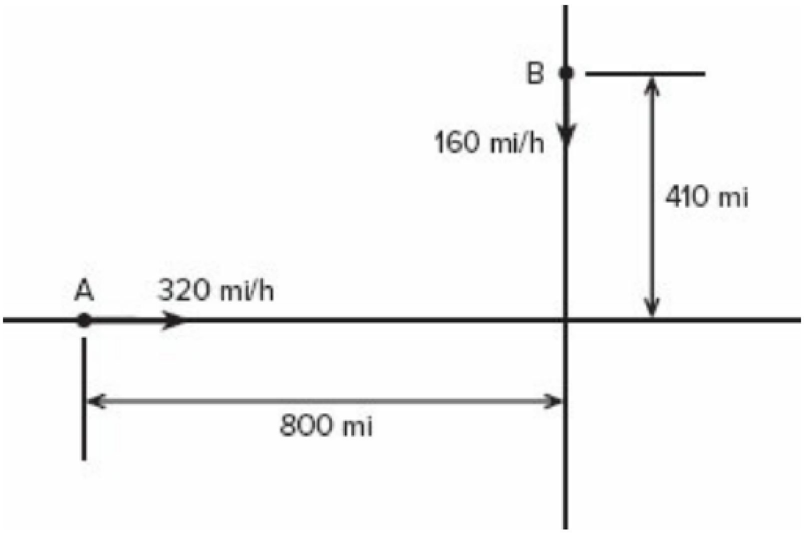
$$x(t) = 4.219(e^{-1.58t} - e^{-6.32t}) \text{ and } v(t) = 26.67e^{-6.32t} - 6.67e^{-1.58t}$$

Plot the displacement and the velocity as a function of time for $0 \leq t \leq 4$ sec. Fit two plots at the top of the window and a plot of both with two vertical axes underneath them. All plots must be of the printing quality.

[3 marks]

Question 10

Aircraft A is flying east at 320 mi/hr, while aircraft B is flying south at 160 mi/hr. At 1:00 p.m. the aircraft are located as shown.



Obtain the expression for the distance *D* between the aircraft as a function of time. Plot *D* versus time until D reaches its minimum value. The plot must be of a printing quality. Use the *roots* function to compute the time when the aircraft are first within 30 mi of each other.

[3 marks]

Question 11

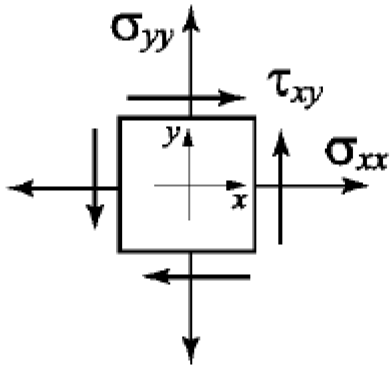
A two-dimensional state of stress at a point in a loaded material in the direction defined by the $x - y$ coordinate system is defined by three components of stress σ_{xx}, σ_{yy} and τ_{xy}. The stresses at the point in the direction defined by the $x' - y'$ coordinate system are calculated by the stress transformation equations:

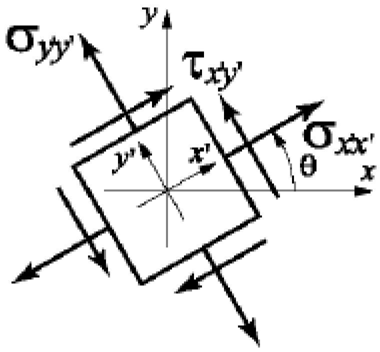
$$\sigma_{x'x'} = \frac{\sigma_{xx} - \sigma_{yy}}{2} + \frac{\sigma_{xx} + \sigma_{yy}}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'y'} = \sigma_{xx} + \sigma_{yy} - \sigma_{x'x'}$$

$$\tau_{x'y'} = -\frac{\sigma_{xx} - \sigma_{yy}}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

where θ is the angle shown in the figure. Write a user-defined MATLAB function that determines the stresses $\sigma_{x'x'}$, $\sigma_{y'y'}$ and $\tau_{x'y'}$ given the stresses σ_{xx}, σ_{yy} and τ_{xy}, and the angle θ. For the function name and arguments, use *[Strain] =StressTrans (S,th)*. The input argument *S* is a vector with the values of the three stress components σ_{xx}, σ_{yy} and τ_{xy} and the input argument *th* is a scalar with the value of θ. The output argument *Strain* is a vector with the values of the three stress components $\sigma_{x'x'}$, $\sigma_{y'y'}$ and $\tau_{x'y'}$.

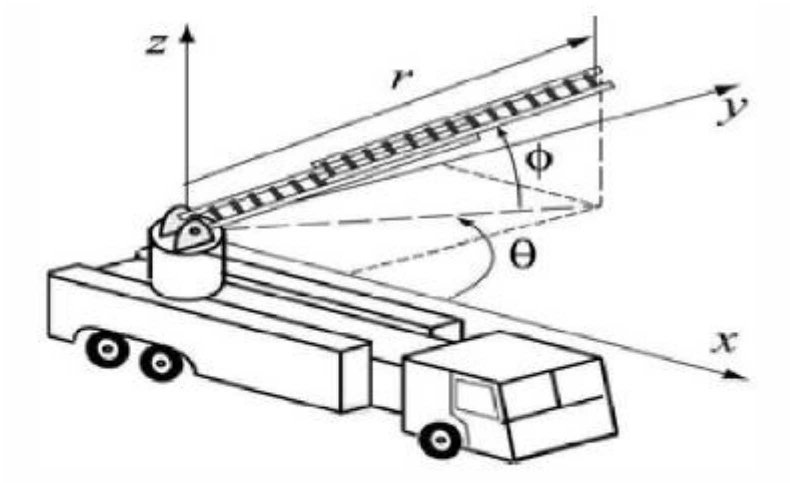




[2 marks]

Question 12

The ladder of a fire truck can be elevated (increase of angle ϕ), rotated about the z axis (increase of angle), and extended (increase of r). Initially the ladder rests on the truck (ϕ = 0 , θ = 0, and r = 8 m). Then the ladder is moved to a new position by raising the ladder at a rate of 5 deg/sec, rotating z at a rate of 8 deg/sec, and extending the ladder at a rate of 0.6 m/sec. Determine and plot the position of the tip of the ladder for 10 seconds.



[3 marks]

Academic skills support
For help and advice on this assessment please contact the assessment setter/s or the module leader. For advice on academic writing and referencing please contact the Faculty of Science, Engineering and Computing (SEC) Academic Success Centre (SASC). Trained staff and students will give you guidance and feedback on assessments. SASC can be contacted by email: SASC@kingston.ac.uk

Submission requirements
Online submission through CANVAS and only in PDF format. Part A should include worked solutions (scans of handwritten notes are permitted) and Part B should include commented MATLAB code and screenshots of the command window and graphs. All graph formatting must be done by code. All programmes should have user-friendly input and formatted output.