**Thapar Institute of Engineering and Technology, Patiala**

**School of Mathematics**

*Assignment Evaluation-II*

**Ph.D. Course work  (Odd Semester 2023-24)**

**Topics: Probability and Distribution theory**

**Subject Name and Code: Research Methodology (DMC007)**

1. The probability that a management trainee will remains with a company is 0.60. The probability that an employee earns more than Rs $10,000$ per month is 0.50. The probability that an employee is a management trainee who remained with the company or who earn more than Rs $10,000$ per month is 0.70. What is the probability that an employee earns more than Rs $10,000$ per month, given that he is a management trainee who stayed with the company.
   Answer- $\frac{2}{3}$

2. In a coin tossing experiment, if the coin shows head, 1 die is thrown and the result is recorded. But if coin shows tail, 2 dice are thrown and their sum is recorded. What is the probability that the recorded number will be 2.
   Answer- $\frac{7}{72}$.

3. A box contains '$a$' white and '$b$' black balls and a second box contains '$c$' white and '$b$' black balls. One ball is transferred from the first box into the second box and one ball is drawn from the second box. Find the probability that it is a white ball.
   Answer- $\frac{a+b+ac+bd}{(a+b)(c+d+1)}$.

4. A box contains at most 5 balls. Two balls are drawn at random and are found to be white. What is the probability of all the balls being white.
   Answer- $1/2$.

5. A box contains 5 red and 4 white balls. Two balls are drawn successively from the box without replacement and it is notice that second one is white. What is the probability that first is also white ?.
   Answer- $\frac{3}{8}$.

6. A and B are two weak student of statistics and their chances of solving a problem in statistics correctly are $\frac{1}{6}$ and $\frac{1}{8}$, respectively. If the probability of their making a common error is $\frac{1}{525}$ and they obtain same answer, find the probability that their answer is correct.
   Answer- $\frac{15}{16}$.

7. Two cards are drawn from a well shuffled desk of 52 cards. Find the probability distribution of the number of aces:
   (i) Successively with replacement.
   (ii) Successively without replacement.

8. Given that
$$f(x) = \begin{cases} \alpha x(1-x), & 0 < x < 1; \\ 0, & \text{otherwise.} \end{cases}$$

   Then find (i) $\alpha$,  (ii) $P(X > 0.5)$,  (iii) cumulative distribution function (c.d.f.),
   (iv) $P\left(X > \frac{1}{2}/X < \frac{1}{4}\right)$,  (v) $P\left(X > \frac{1}{2}/X > \frac{1}{4}\right)$.

9. Define Binomial distribution and find mean, variance and moment generating function (m.g.f.).

10. A coin is biased so that a head is twice as likely to appear as a tail. If the coin is tossed 6 times, find the probability of getting (i) exactly 2 heads, (ii) at least 3 heads, (iii) at most 3 heads.

11. It is known that 5% of the books bounds at a certain bindery have defective bindings. Find the probability that 2 of the 100 books bound by this bindery will have defective bindings.

12. For a Binomial distribution, the mean 6 and standard deviation $\sqrt{2}$. Find first two terms of the distribution.

13. A fair die is thrown and an outcome of 4 or 5 is success. If the die is thrown 9 times and $X$ denotes the number of success. (i) Find the mean and variance, (ii) $P(X = 2)$, (iii) $P(2X - 3 = 2)$, (iv) $P(X \leq 2)$.

14. Six dice are thrown 729 times. How many times do you expect at least 3 dice to show 5 or 6.
Answer- 233.

15. Comment of the statement: The mean of a Binomial distribution is 5 and standard deviation is 3.

16. A perfect coin is tossed twice. Find the m.g.f. of the number of heads. Hence find the mean and variance.

17. 17. A boy is throwing stones at a target, what is the probability that his $10^{th}$ throw is his $5^{th}$ hit, if the probability of hitting the target at any trial is 0.05.
Answer- $126 \times \frac{19^5}{20^{10}}$

18. A pair of fair dice is thrown 5 times. If getting a doublet is considered to be success. Find the probability of getting:
(i) at least 2 success,
(ii) at most 2 success,
(iii) exactly 2 failure.

19. Define the poisson distribution. Also find its mean, variance and moment generating function.

20. Prove that poisson distribution is the limiting case of binomial distribution under the following conditions:

   (a) $n \to \infty$,
   (b) $p \to 0$,
   (c) $np = \lambda$ (finite).

21. If 5% of the electric bulbs manufactured by a company are defective, find the probability that in a sample of 100 bulbs,

   (a) none is defective,

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(b) 5 bulbs will be defective.

Answer-(i) $e^{-5}$  (ii) $\frac{e^{-5} 5^5}{5!}$.

22. If the probability that an individual suffers a bad reaction from an injection of a given serum is 0.001, determine the probability that out of 2000 individuals,

    (a) exactly 3 suffer a bad reaction,
    (b) more than two individuals, will suffer a bad reaction.

    Answer- (i) $\frac{4}{3} e^{-2}$  (ii) $1 - 5 e^{-2}$.

23. A small voting district has 101 female and 95 male voters. A random sample of 10 voters is drawn. What is the probability that exactly 7 of the voters will be female ?
    Answer- 0.13.

24. A group of 10 individuals are used for biological test with the following blood types, type O-3 people, type A-4 people and type B-3 people. What is the probability that a random sample of 5 people will contains 1-type O, 2-type A and 2-type B ?
    Answer-$\frac{3}{14}$.

25. Define the uniform/rectangular distribution. Also find the cumulative distribution function (C.D.F.), mean and variance.

26. If $X$ is uniformly distributed in $[-2, \ 2]$, then find the $P(X < 0)$ and $P\left(|X - 1| \geq \frac{1}{2}\right)$ using P.D.F. and C.D.F. approach.
    Answer- (i) $\frac{1}{2}$,  (ii) $\frac{3}{4}$.

27. Define the exponential distribution. Also find its mean, variance and moment generation function.

28. The time (in hours) required to repair a machine is exponentially distributed with parameter $\frac{1}{3}$. What is the probability that the repair time exceeds 3 hours?.
    Answer- $\frac{1}{e}$.

29. The life length (in months) of an electric component follows an exponential distribution with parameter $\frac{1}{2}$. What is the probability that the component survives at least 10 months, given that already it had survived for more than 9 months?
    Answer- $e^{-\frac{1}{2}}$.

30. Define the normal distribution. Also find its mean, variance and moment generating function for the random variable $X$.

31. The saving bank account of a customer showed an average balance of $ 150 and standard deviation of $ 50. Assuming that the account balance(s) are normally distributed, find the percentage of the account(s)  (i) over $ 200,   (ii) between $ 120 and $ 170. Given that $P(0 < Z < 1) = 0.3413$, $P(Z < 0.4) = 0.6554$ and $P(Z < -0.6) = 0.2743$.
    Answer- (i) 0.1587,  (ii) 0.3811.

32. In normal distribution, 31% of the items are under 45 and 8% are over 64. Find the mean and standard deviation. Given that $P(Z < -1.4) = 0.08$ and $P(Z > 0.5) = 0.31$.
    Answer- $\mu = 50$ and $\sigma = 10$.

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