Project
Stochastic Simulation Techniques and Structural Reliability

Results have to be submitted in a project report (PDF)
The quality of the report (such as the quality of figures, or the overall structure) will also be evaluated. The report should include conclusive scientific interpretations of the results. The used MATLAB codes to be added as appendies at the end of the PDF report and to be referenced in the text.

1 Task01: Random Fields (10 Points)

- Please download the Finite Element Method (FEM) Package CALFEM 3.4. The package provides a well implemented academic Finite Element Environment for MATLAB.
- Study the example ed1.m (Modal analysis of frame). Understand the code and run the example.
- Consider the material properties (i.e., the E-modulus) of the frame elements are random with

$$\mu_X = 250.0\ GPa,\ \sigma_X = 10\ GPa,\ \Theta = 2.5$$

- Generate a random field of length 4, and do a Monte Carlo Analysis to study the stochastic effect of the E- Young's modulus on the first eight eigenfrequencies and mode shapes of the steel frame?
- Now, document your solution in the PDF report and include your Matlab code as an appendix in the same report.

2 Task02: Random Fields (10 Points)

- Please download the Finite Element Method (FEM) Package CALFEM 3.4. The package provides a well implemented academic Finite Element Environment for MATLAB.
- Study the example exs6.m (Analysis of a plane frame). Understand the code and run the example.
- Calculate the random fields the frame elements after dividing each of them into two, cosidering the E-modulus of the frame elements as random with $\mu_X = 210.0\ GPa,\ \sigma_X = 15\ GPa,\ \Theta = 6$.
- Now assign the calculated random material properties and examine the effect of the random material properties on the horizontal displacements at the top of the frame and at the center of the horizontal part of the frame?
- Document your solution in the PDF report and include your Matlab code as an appendix in the same report.

3 Task03: Random Fields (10 Points)

- Please download the Finite Element Method (FEM) Package CALFEM 3.4. The package provides a well implemented academic Finite Element Environment for MATLAB.
- Study the example exs5.m (analysis of a simply supported beam). Understand the code and run the example.
- We adapt now the code by dividing the beam into 6 finite elements, and say that again the material properties (E- Young's modulus) of the 6 elements are random with

$$\mu_X = 200.0\ GPa,\ \sigma_X = 10\ GPa,\ \Theta = 4.$$

- Generate a random field of length 6. Assign the calculated random material properties to the element and repeat the computations by applying a single load of 5 kN.
- What is the probability that the vertical displacement at the point of loading will exceed 12 mm? Assuming that a displacement larger 12 mm would lead to failure, what is the corresponding reliability index?
- To which values does the standard deviation of the field needs to be changed that the reliability index is at least 3.0? (try to find out by repeated MCS).

- Now, document your solution in the PDF report and include your Matlab code as an appendix in the same report.

4 Reliability Analysis (10 Points)

Given the two limit state functions:

$$g_1 = 20 + X_1^3 - 5X_2^2,$$

$$g_2 = 2 + 2X_1 + X_2,$$

We assume for g_1 that X_1 and X_2 are two independent R.V. following both normal distribution such that: $X_1 \sim \mathcal{N}(1, 0.2)$ and $X_2 \sim \mathcal{N}(1, 0.5)$. For g_2, we assume that X_1 and X_2 are independent random variable. X_1 follows a normal distribution such that: $X_1 \sim \mathcal{N}(1, 0.3)$ and X_2 is a standardized Gaussian R.V. For the two limit state functions (g_1 and g_2):

- Evaluate (with Matlab) the reliability index β and the probability of failure P_f while applying MCS and FOSM Method. Vary the number of samples N and conclude its effect.

- Plot the limit state functions and the safe/failure regions with MCS in the original space (X-space) and U-space.

- Evaluate (with Matlab) β and P_f with FORM method. Compare the results to what you got previously and give your comments.

- Plot the limit state functions and P_f with the FORM method in U-space.

- Evaluate the most probable failure point (MPFP) in X-space and U-space.

- Conduct now the calculation with UQLab for both functions (apply the MCS and FORM method) and compare this result to the previous ones. Hint: make a table for the results of β and P_f you got with all methods.

5 Reliability Analysis (10 Points)

- While implementing the FOSM method, some limit functions give failure probability values different from the FORM and MCS results. Cite three different factors that lead to such difference.

- We have the following limit state function:

$$g_2(X_1, X_2) = X_1 - 2X_2$$

The random variables X_1 and X_2 are uncorrelated. $X_1 \sim \mathcal{N}(\mu_1 = 1, \sigma_1 = 0.4)$ and X_2 follows a log-normal distribution such that $X_2 \sim \mathcal{LN}(\mu_2 = 1.5, \sigma_2 = 0.2)$

  - Use UQlab to evaluate the reliability index β and the probability of failure P_f while applying the following methods (Hint: please take a look at the script "uq_Example_Reliability_01_RS.m" (from the "Examples" in "Reliability" folder)):
    - Monte Carlo simulation,
    - First-order reliability method (FORM)
    - Importance sampling (IS)
    - Adaptive Kriging-Monte Carlo Simulation (AK-MCS)
  - Compare the results and give interpretations.

You can download the open-source framework UQLab, and you need to register. Refer to the user manual 1, and user manual 2 to get more information on how to apply the framework.