HW1: Linear Algebra Review

1. Find all solutions of

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 0 & 1 & 0 & 1 \end{bmatrix} x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \text{ where } x \in R^4$$

Which is the minimum norm one?

2. Find all minimizers of the 2-norm of

$$\begin{bmatrix} 1 & 0 \\ 2 & 1 \\ 3 & 2 \\ 4 & 3 \end{bmatrix} x - \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \end{bmatrix}, \text{ where } x \in R^2$$

3. If A is symmetric, what is the relationship between its eigenvalues and singular values?

4. For $a, b \in R^n$, show that $\det(I + ab^T) = 1 + a^T b$.

5. Show that $\max |a_{ij}|$ is a norm in the space of n x m matrices, but not an induced norm.

6. For the matrix $\begin{Bmatrix} 1 & 0 & 2 \\ 2 & 1 & 4 \\ 3 & 2 & 6 \\ 4 & 3 & 8 \end{Bmatrix}$, compute the SVD and specify an orthonormal basis for its null and range spaces.

7. SVD application in the modeling of a noisy oscillatory signal as the output of an autoregressive model:

y(n)=[y(n-1),y(n-2),...]*q

Define the signals:

```
>> t=(0:.01:10)';        % time
>> n=rand(size(t))-.5;   % noise
>> y=sin(10*t+1)+.02*n;  % measurement
>> N0=10:length(t);      % fitting window
```

Optional Filtering for frequency-weighted fit:

```
>> F = tf(.01,[1 -.99],.01); y = lsim(F,y);
```

Form the regressor vector by taking lags of the output:

```
>> W=[y(N0-1),y(N0-2),y(N0-3),y(N0-4),y(N0-5),y(N0-6),y(N0-7),y(N0-8),y(N0-9)];
```

Compute the least squares fit:

```
>> q=W\(y(N0))
>> plot(t,y,t(N0),W*q,t(N0),W*q-y(N0)); pause    % check the fit
```

Form the autoregressive model transfer function: we expect a resonance at the oscillation frequency

```
>> g=tf(1,[1 -q'],.01)
>> bode(g)              % check the t.f.
```

Questions:

1. What is the expected order of the model? How many lags/columns of W do you need?

   ```
   >> s=svd(W)          % svd of regressor
   >> s2=svd(W'*W)      % svd of gramian
   ```

2. What is the relationship between s and s2?
3. The singular values of W appear to reach a floor related to the noise. Derive this value analytically and verify with an example.
4. Experiment with the noise amplitude and model order and describe their effect.
5. Experiment with a signal composed of two frequencies, say 10 and 2, and comment on your observations.