Assessment: **Assignment 1**
Due: **24 August 2023**
Marks: **200**
Value: **20%**

1 (worth 200 marks)

Introduction

Periodic functions are widely encountered in mathematics, physics and engineering; they are used to describe the behaviour of phenomena that exhibit periodicity such as vibrations, oscillations and waves. A periodic function $f(x)$ with the period 2π can be approximated in terms of trigonometric functions:

$$f(x) = c_1 + \sum_{k=1}^{\frac{N-1}{2}} \left(c_{2k}\cos(kx) + c_{2k+1}\sin(kx)\right), \tag{1}$$

where N is an odd number that is greater than 3, and $(c_1, c_2, ..., c_N)$ is a set of the coefficients, which are unknown. From a given set of N data points: (x_i, f_i), $i = (1, 2, \ldots, N)$, the coefficients can be found by solving the algebraic equation set: $f_i = f(x_i)$ with $i = (1, 2, \ldots, N)$. When the coefficients are found, one can use (1) to estimate f at any value of x.

Taking $N = 3$, for example, equation (1) reduces to

$$f(x) = c_1 + c_2\cos(x) + c_3\sin(x). \tag{2}$$

Given a set of 3 data points: (x_i, f_i), $i = (1, 2, 3)$, the following algebraic equation set can be established

$$\mathcal{A}\mathbf{c} = \mathbf{f}, \tag{3}$$

where

$$\mathcal{A} = \begin{bmatrix} 1 & \cos(x_1) & \sin(x_1) \\ 1 & \cos(x_2) & \sin(x_2) \\ 1 & \cos(x_3) & \sin(x_3) \end{bmatrix}, \mathbf{c} = \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix} \text{ and } \mathbf{f} = \begin{pmatrix} f_1 \\ f_2 \\ f_3 \end{pmatrix}.$$

This system can be solved for the three coefficients: (c_1, c_2, c_3). It is noted that $\mathcal{A}$ in (3) is called the coefficient matrix, and in general $\mathcal{A}$ is the matrix of size N-by-N, and $\mathbf{c}$ and $\mathbf{f}$ are vectors of length N.

In this assignment, a set of data points in the period of 0 to $L = 2\pi$ are given in File `<A1_input.txt>` and the resulting algebraic equation set is solved by using Matlab function `<mldivide>` (i.e. $\mathbf{c} = \mathcal{A} \setminus \mathbf{f}$). Your tasks are to construct a periodic analytic function f representing these data points by means of (1), and then utilise it to estimate values of f at 100 equispaced values of x: $x_1 = 0, x_2, \ldots, x_{100} = L$.

Requirements

For this assessment item, you must perform hand calculations, where there are only 3 given data points:

1. Construct $f(x)$ using the first, fourth and last data points of the data set in the file.
2. Estimate f at $x = L/4$ and $x = L$. Report the function values rounded to four significant figures.

You must also produce MATLAB code which:

3. Repeats Requirements 1 and 2. Verify the answers by using the reported results from Requirement 2.
4. Loads File `<A1_input.txt>` into MATLAB. Verify that the load has been successful by comparing the obtained first and last rows of the numeric data part with those from copy-and-paste.
5. Constructs $f(x)$ using all data points in the file, where loops are used to form the coefficient matrix $\mathcal{A}$. Display the resulting coefficients in the command window.
6. Constructs $f(x)$ using all data points in the file, where the coefficient matrix $\mathcal{A}$ is formed without using any loops (vectorised code). Verify the calculations by comparing the obtained matrix $\mathcal{A}$ with that from Requirement 5. Display the resulting coefficients in the command window.
7. Estimates values of f at 100 equispaced values of x: $x_1 = 0, x_2, \ldots, x_{100} = L$ using the coefficients from Requirement 5 or Requirement 6. Display the three values f_{20}, f_{40} and f_{60} in the command window.
8. Plots the given data (from the file) and the estimated data from Requirement 7 on the same graph, where the x and y axes are used to represent the variables x and f, respectively.
9. Produces an ASCII file that stores the values of x and the approximate values of f from Requirement 7, where the values are rounded to five decimal places for the variable x and four significant figures for the variable f.
10. Has appropriate comments throughout.

You must submit pseudocode for Requirement 5.
You must submit a short video (10-30 seconds) where you discuss:

- the part of the code of which you are most proud; and
- the part of the code which you found most difficult to get working correctly.

Your video must show the relevant part(s) of the code while you are discussing them. Your video will be marked based on whether one is submitted or not: full marks for this item if a video is submitted; zero marks if a video is not submitted. Your video should not be in full HD format (i.e. your video should be no more than 1280 pixels across), so that the file size does not become too large. There is a lot of verification involved in this assignment. This is to help you: by checking your answers, you know if what you are doing is correct (you should never trust the answer supplied by a computer!).

Assessment Criteria

Your submission will be assessed using the following scheme. Note that you are marked based on how well you perform for each category, so the correct answer determined in a basic way will receive half marks and the correct answer determined using an excellent method/code will receive full marks.

Quality of header(s) and comments	15 marks
Quality of reporting of the final results in Command Window	15 marks
Quality of Requirements 1-2	20 marks
Quality of Requirement 3	10 marks
Quality of Requirement 4	25 marks
Quality of Requirement 5	25 marks
Quality of Requirement 6	25 marks
Quality of Requirement 7	25 marks
Quality of Requirement 8	15 marks
Quality of Requirement 9	15 marks
Quality of pseudocode	5 marks
Submission of video	5 marks

Troubleshooting

You can access MATLAB over the web through MATLAB Online if you are having trouble running Matlab on your personal computer.

Submission

Submit your Matlab program, along with input/output files, by the due date to the StudyDesk. The Matlab program includes a single `m` file that covers all relevant requirements and its associated function files (if existed). Submit your hand calculations and your pseudocode as a pdf file, and your video as a standard video file. If your video is too big, convert it to a lower resolution using Handbrake (https://handbrake.fr/) or something similar. The maximum submission size is 5 MB. Note that

- You do not need to rename your files when uploading: the system automatically segregates different students' submissions.
- If you can see that the files have uploaded, then you have successfully submitted your assignment. There is no need to click a "send for marking" button, but you will have to click a button confirming that the submission is your own work.
- You MUST upload all of your code, your pdf file and your video file along with input/output files in a *.zip file. You do not need to install any special software to be able to do this. In Windows: right-mouse-click on the file(s)/folder(s) that you want to include, then choose "Send to" → "Compressed (zipped) folder". The following are the only file types that can be submitted:
  - *.zip
  - *.pdf
  - *.doc

  - *.docx
  - standard video formats

  The system will block any attempt by you to upload a file which doesn't match any of those file extensions.

- After the submission deadline, if you forgot to submit a file, do not upload it after the due date: the submission time is based on when the last file was uploaded. You should email the examiner in this circumstance (with any file attached).