Math 232 – Computing Assignment 2

Due Date: Friday, June 23th, at 11:59pm.

You must upload to Crowdmark both your code (as a .pdf file) (to Code - Computing Assignment 2) and your report (to Report - Computing Assignment 1). If Crowdmark indicates that you submitted late, you will be given 0 on the assignment. No requests for extensions or marks for late assignments will be considered. Ideal length for your computing report must be exactly 2 pages. There will be 2 marks penalty for each extra page you use. There will be a document uploaded -'Computing BFS' on lecture schedule page on canvas for a small problem that would be helpful for this assignment. **Note that some parts of this assignment will not be marked and you will receive 1 mark for completing all assignment as 'completion marks'(Only if a good attempt is made to write the solution for the questions considered for completion. Only writing answers without any explanation will not be considered as complete.)**.

- Keep in mind that Canvas discussions are open forums.
- Acknowledge any collaborations and assistance from colleagues/TAs/instructor.

Programming Preamble: *Matlab*: inv(A) finds the inverse matrix of matrix A.

Matlab: You need to be able to enter column vectors and matrices.

Matlab: cat(2,A,B) concatenation of A and B (use to produce an augmented matrix).

Computing Assignment

Required submission: 2 page PDF report and Matlab or Python code (.m or .py respectively, exported as a .pdf) uploaded to Canvas..

1. Consider the matrix A as: $\begin{bmatrix} 1 & 2 & -1 & 3 & 3 \\ 1 & 0 & -1 & 2 & 1 \\ 1 & -1 & 1 & -1 & 0 \end{bmatrix}$ and $\mathbf{c}=\begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$ and $\mathbf{b}=\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$ Further, consider the linear program

$$\begin{aligned} &\text{Maximize } \mathbf{cx} \\ &\text{Subject to } A\mathbf{x} = \mathbf{b} \\ &\qquad \mathbf{x} \in \mathbb{R}^5 \\ &\qquad x_i \geq 0 \quad \forall i = 1, 2, \ldots 5. \end{aligned}$$

Here, we are interested in computing a vector $\mathbf{x}$ such that it maximizes the function $\mathbf{cx}$. We learnt many methods for computing solution to the system of linear equations in this course. Here we will use inverses(section 3.5 and concepts of linear independence). Following questions will give you a way to approach this question. You may write it as a report while answering the following questions.

a) Show that the system of equations A$\mathbf{x}$=$\mathbf{b}$ is consistent.

b) Show that this system A**x**=**b** has infinitely many solutions.

c) In the real world, we come across such problems where we have infinitely many solutions and we want to find the best one corresponding to given function **cx**. To reach the best solution, we find the set of good solutions for the given problem which are considered as candidates to be best. Such solutions are known as basic feasible solutions. For a mxn matrix, Choose a set of m column vectors and create a new matrix B(that will be mxm matrix). Let set β represents the indices of column vector chosen.

$$x_i = \begin{cases} B^{-1}b_i \text{ for all } i \in \beta \\ 0 \text{ otherwise.} \end{cases}$$

The above solution is known as basic solution. And if $x_i \geq 0 \quad \forall i = 1, 2, \ldots n.$ also, the solution is called basic feasible solution(BFS). If B^{-1} does not exist or if $x_i < 0 \quad$ for some $i = 1, 2, \ldots n.$, the corresponding solution will not be a BFS. For a mxn matrix, there can be at most $\binom{m}{n}$ BFS i.e(10 in above case) So you need to inspect all cases and show your work.

Find all the basic feasible solutions of the above system of equations Ax=b using MATLAB. Include your output from MATLAB for all solutions. In case some solution is not BFS conclude why it is not BFS.

d) Among the BFS found in part (c), which one is the best.(To find the best solution, compute the **cx** for each BFS **x** and find the largest value as we are maximizing.)

e) Compute the best solution if you want to minimize instead of maximizing the objective function **cx**.

f) Consider a set α which consists of collection of all sets β which gives us BFS. If you change the vector **b** to

$$\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

. Will you get the BFS corresponds to all sets in α again? Elaborate your answer properly.

g) Compute the basis of row space, null space, left null space and column space of A using MATLAB.

h) Assume that a furniture company manufactures tables, chairs and dressers using two kinds of wood, walnut wood and oak wood. Suppose that each table, chair and dresser need 3, 2 and 6 units of walnut wood respectively and 2, 4 and 1 unit of oak wood respectively. Assume that the total 400 units of walnut wood and 300 units of oak wood is available per week. If profit from selling each table is CAD 30, each chair is CAD 20 and each dresser is CAD 15. How many units of each table, chair and dresser should be manufactured and sold in order to maximize the profit of the company? (You can round off your solution to the nearest integer) Compute the all BFS of the system and then find the best one.