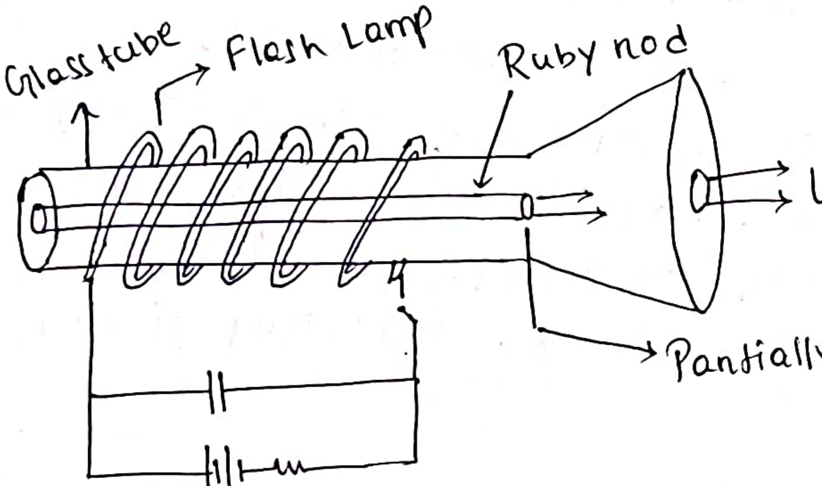
Ruby Laser:

Construction:

→ Ruby laser is a 3 level laser system.

→ Ruby chemically consists of Al_2O_3 in which a small percentage of Al^{3+} ion has be replaced (doped) by Cr^{3+}.

→ Ruby laser was made from a single cylindrical crystal of ruby whose ends were flat, with one of the ends completely silvered & the other partially silvered.



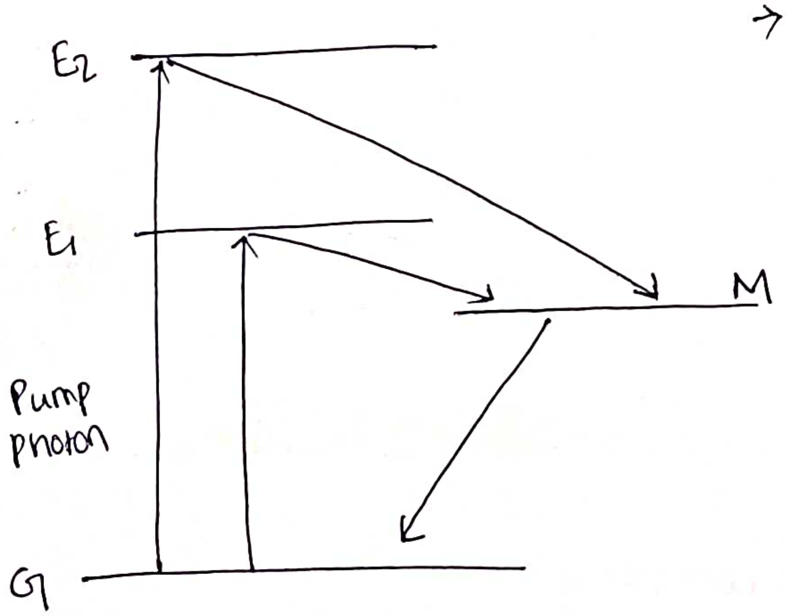
* The Al_2O_3 crystal which serves as a medium to suspend the Cr ions in known as the host crystal.

→ Characteristics of host crystal affect the laser action.

→ Cr ions are known as activator atoms.

→ Amout of doping is approximately 0.05 wt % of Cr_2O_3

→ Simplified energy level diagram of Cr ion in ruby laser.



→ The main characteristics of the energy levels of a Cr ion is the fact that the bands E_1 & E_2 have a lifetime of $\lesssim 10^{-8}$ sec whereas the state marked M has a lifetime of $\sim 3 \times 10^{-3}$ sec

A state characterised with such a long life is known as metastable state.

Working:

→ The Cr ion in ground state can absorb a photon & make a transition to the energy band E_1. This optical pumping phenomenon & the photons which are absorbed by the Cr ions are produced by the flash lamp. ~~These~~

→ These photons can absorbs photons of lower wavelength & move to energy band E_2.

→ In both cases, it makes a non-radiative transition immediately (in time $\lesssim 10^{-8}$ sec) to the metastable state M.

→ Since, the state M has a very long life ($\sim 10^{-3}$ sec), the no. of atoms in this state keeps on increasing & population inversion occurs between states M & G.

Once population inversion occurs, light amplification can take place.

Drawbacks:

→ Require high pump intensity

→ overall efficiency is very low

Main components:

→ Flash lamp

→ Ruby crystal

→ Glass tube

→ Power supply.

Transverse nature of em waves:

A plane wave is that whose amplitude of vibration is same at any point in a plane perpendicular to the specified direction. The soln. of the wave eqn:

$$\nabla^2 \psi - \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2} = 0 \text{ is}$$

$$\psi = \psi_0 e^{-i(\omega t - \vec{k}\cdot\vec{r})}$$

The soln. of the electromagnetic wave eqn:

$$\nabla^2 \vec{E} - \frac{1}{c^2}\frac{\partial^2 \vec{E}}{\partial t^2} = 0 \;\&\; \nabla^2 \vec{H} - \frac{1}{c^2}\frac{\partial^2 \vec{H}}{\partial t^2} = 0$$

are:

$$\vec{E} = \vec{E}_0 e^{-i(\omega t - \vec{k}\cdot\vec{r})} \;\&\; \vec{H} = \vec{H}_0 e^{-i(\omega t - \vec{k}\cdot\vec{r})}$$

where $\vec{E}_0$ & $\vec{H}_0$ are the complex amplitudes which are constant in space & time.

$\vec{k}$ is the propagation vector given by:

$$\vec{k} = k\hat{n} = \frac{2\pi}{\lambda}\hat{n} = \frac{2\pi}{c/\nu} = \frac{2\pi\nu}{c} = \frac{\omega}{c}\hat{n}$$

$\hat{n}$ is a unit vector in the direction of propagation of the wave.

Now,

$$\vec{\nabla}\cdot\vec{E} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left(\vec{E}_0 e^{-i(\omega t - \vec{k}\cdot\vec{r})}\right)$$

$$= \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\cdot\left\{\left(E_{0x}\hat{i} + E_{0y}\hat{j} + E_{0z}\hat{k}\right)\left(e^{i(k_x x + k_y y + k_z z)}\right)\left(e^{-i\omega t}\right)\right\}$$

$$\vec{\nabla}\cdot\vec{E} = i\left(E_{0x}k_x + E_{0y}k_y + E_{0z}k_z\right) e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$= i\left\{(\hat{i}E_{0x} + \hat{j}E_{0y} + \hat{k}E_{0z})\cdot(\hat{i}k_x + \hat{j}k_y + \hat{k}k_z)\right\} e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$= i(\vec{E}_0\cdot\vec{k})e^{i(\vec{k}\cdot\vec{r} - \omega t)}$$

$$= i\vec{k}\cdot\vec{E}$$

$$\boxed{\vec{\nabla}\cdot\vec{E} = i\vec{K}\cdot\vec{E}}$$

Similarly : $\cancel{\vec{\nabla}\cdot\vec{B} = i\vec{K}\cdot\vec{B}}$ $\boxed{\vec{\nabla}\cdot\vec{H} = i\vec{K}\cdot\vec{H}}$

$$\boxed{\vec{\nabla} \approx i\vec{K}}$$

Now, $\dfrac{\partial\vec{E}}{\partial t} = \dfrac{\partial}{\partial t} E_0 e^{-i(\omega t - \vec{K}\cdot\vec{r})} = -i\omega\vec{E}$

$$\boxed{\frac{\partial}{\partial t} \approx -i\omega}$$

So, the maxwell eqns in free space in terms of $(i\vec{K})$ & $(-i\omega)$ operators can be written as:

$$\vec{K}\cdot\vec{E} = 0 \quad - (1)$$
$$\vec{K}\cdot\vec{H} = 0 \quad - (2)$$
$$\vec{K}\times\vec{E} = \mu_0\omega\vec{H} \quad - (3)$$
$$-\vec{K}\times\vec{H} = \varepsilon_0\omega\vec{E} \quad - (4)$$

from eqn (1) & (2) we can write:

$\vec{E} \perp \vec{K}$ & $\vec{H} \perp \vec{K}$

from (3) we can write:

$\vec{H} \perp$ to $\vec{K}$ & $\vec{E}$

from (4) we can write:

$\vec{E} \perp$ to $\vec{K}$ & $\vec{H}$

from (1), (2), (3), (4) we can write.

$\vec{E}$ & $\vec{H}$ are $\perp$ to $\vec{K}$

Thus, em waves are transverse in nature

$\vec{E}$, $\vec{H}$ & $\vec{K}$ are all orthogonal.

Impedence:

$$\vec{K} \times \vec{E} = \mu_0 \omega \vec{H}$$

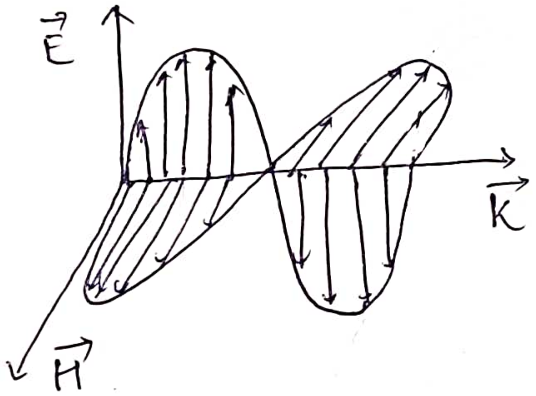
$$\vec{H} = \frac{k}{\mu_0 \omega} (\hat{n} \times \vec{E}) \Rightarrow \vec{H} = \frac{1}{c\mu_0} (\hat{n} \times \vec{E}) \quad (\because K = \frac{\omega}{c})$$

$$\Rightarrow \vec{H} = c\,\varepsilon_0 (\hat{n} \times \vec{E}) \quad (\because c^2 = \frac{1}{\mu_0 \varepsilon_0} \Rightarrow c\,\varepsilon_0 = \frac{1}{c\mu_0})$$

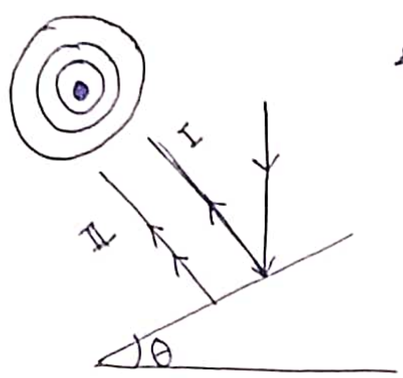
$$\Rightarrow \left| \frac{\vec{E}}{\vec{H}} \right| = \frac{E_0}{H_0} = \frac{1}{c\varepsilon_0} = \sqrt{\frac{\mu_0}{\varepsilon_0}} = Z_0$$

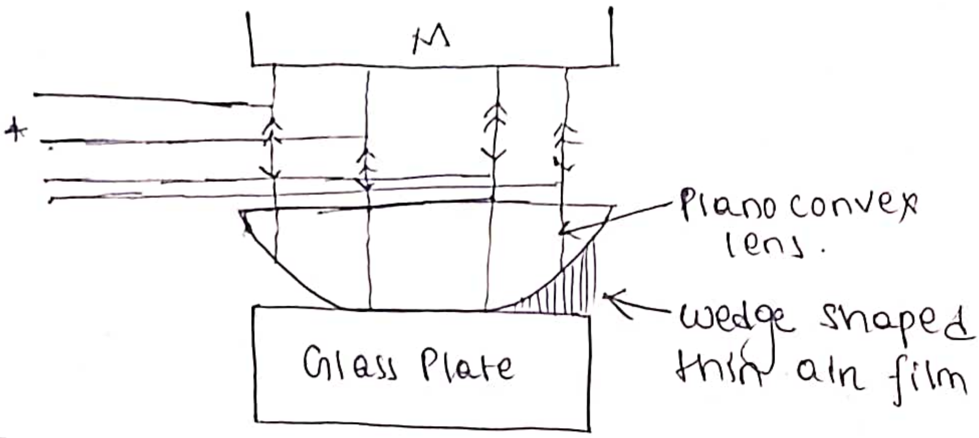
Here 'Z_0' has the dimensions of electrical resistance. (impedance)

$\therefore$ Z_0 is called impedance of vaccum.



Newton's Ring:





A plano convex lens is placed on a glass plate in such a way that a ~~thin wedge shape~~ its curved surface lies on the glass plate. Then an air film of gradually increasing thickness is formed between the 2 surfaces.

When a beam of monochromatic light is allowed to fall normally on this film & viewed, alternate dark & bright circular fringes are observed. These circular fringes are formed due to interference between the rays reflected from upper surface & lower surface of the air film formed between the curved surface and glass plate.

These fringes are circular due to the circular geometry of the air film.

The thickness of air film corresponding to each fringe is same throughout the circle.

It's observed by Newton therefore called Newton's ring.

Path Difference between I & II,

$$\boxed{\Delta = 2\mu t \cos(r+\theta) + \lambda/2}$$

where,

$r \rightarrow$ angle of refraction

$\theta \rightarrow$ Angle of wedge.

For normal incidence, $r = 0$

& Since, the radius of curvature of the lens is very large the ~~of~~ angle of wedge is very small,

$$\Rightarrow \theta \approx 0$$

$\therefore$ Path difference $\Rightarrow$

$$\boxed{\Delta = 2\mu t + \frac{\lambda}{2}}$$

At point of contact, $t = 0$

$\Rightarrow \Delta = \frac{\lambda}{2} \rightarrow$ satisfies the condition of destructive interference.

Hence, central spot (ring) is dark due to reflected ray.

Path diff = even multiple of $\frac{\lambda}{2} \rightarrow$ constructive interference

Path diff = odd multiple of $\frac{\lambda}{2} \rightarrow$ destructive interference.

<u>Condition for Constructive Interference</u>:

Path difference $= 2n\frac{\lambda}{2}$

$$2\mu t \pm \frac{\lambda}{2} = 2n\frac{\lambda}{2}$$

$$\boxed{2\mu t = (2n \pm 1)\frac{\lambda}{2}}$$

↳ Constructive Interference (Bright Ring)

<u>Condition for Destructive Interference</u>:

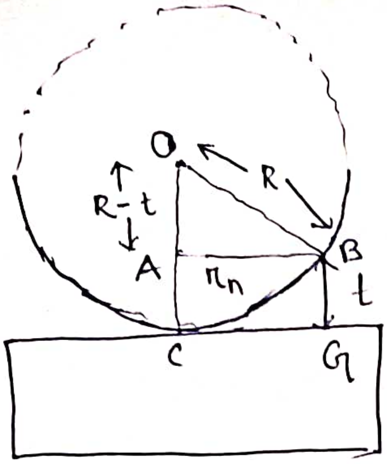
Path difference $= (2n \pm 1)\frac{\lambda}{2}$

$$2\mu t \pm \cancel{\frac{\lambda}{2}} = (2n \pm \cancel{1})\frac{\lambda}{2}$$

$$\Rightarrow 2\mu t = 2n\frac{\lambda}{2}$$

$$\Rightarrow \boxed{2\mu t = n\lambda}$$

↳ Destructive Interference (Dark Ring).



Let: ~~OA~~ $OB = R$; $AC = BG = t$

$OA = R - t$

$AB = r_n$

* Let R be the radius of curvature of the lens.
* let at point G, the thicknes of the air film $= BG = t$
* The respective ring radius $= AB = r_n$

from ΔOAB,

$$R^2 = (R-t)^2 + r_n^2$$

$$\cancel{R^2} = \cancel{R^2} - 2Rt + \cancel{t^2} + r_n^2$$

since, the thickness of the wedge shape air film is very small $\Rightarrow t^2 \rightarrow$ neglected.

$$\Rightarrow r_n^2 = 2Rt$$

$$\Rightarrow \boxed{t = \frac{r_n^2}{2R}}$$

thickness of air film

Condition for Constructive Interference:

$$2\mu t = (2n \pm 1)\frac{\lambda}{2}$$

$$\Rightarrow \cancel{2}\mu \cdot \frac{r_n^2}{\cancel{2}R} = (2n+1)\frac{\lambda}{2} \Rightarrow r_n^2 = (2n+1)\frac{\lambda R}{2\mu}$$

For Air, $\mu = 1$:

$$r_n^2 = (2n+1)\frac{\lambda R}{2} \Rightarrow \boxed{r_n = \sqrt{(2n+1)\frac{\lambda R}{2}}}$$

$n = 0, 1, 2, 3 \ldots$

1'st ~~Bright~~ ring for $n=0$:

$$r_1 = \sqrt{\frac{\lambda R}{2}} \Rightarrow r_1 \propto \sqrt{1} \quad (n=0)$$

$$r_2 = \sqrt{\frac{3\lambda R}{2}} \Rightarrow r_2 \propto \sqrt{3} \quad (n=1)$$

$$r_3 = \sqrt{\frac{5\lambda R}{2}} \Rightarrow r_3 \propto \sqrt{5} \quad (n=2)$$

Radii of bright rings proportional to square root of odd natural no.'s.

Condition for Destructive Interference:

$$2\mu t = n\lambda$$

$$\cancel{2}\mu \frac{r_n^2}{\cancel{2}R} = n\lambda \Rightarrow r_n^2 = \frac{n\lambda}{\mu}$$

For air, $\mu = 1$:

$$r_n^2 = \frac{n\lambda}{} \Rightarrow r_n = \sqrt{n\lambda}$$

$n = 1, 2, 3 \ldots$

$r_1 = \sqrt{\lambda R} \Rightarrow r_1 \propto \sqrt{1}$ $(n=1)$

$r_2 = \sqrt{2\lambda R} \Rightarrow r_2 \propto \sqrt{2}$ $(n=2)$

$r_3 = \sqrt{3\lambda R} \Rightarrow r_3 \propto \sqrt{3}$ $(n=3)$

~~Radii~~ Radii of Dark ring proportional to the square root of natural nos.

i) Determination of wavelength of the monochromatic source:

if D_n be the diameter of the n^{th} dark ring then:

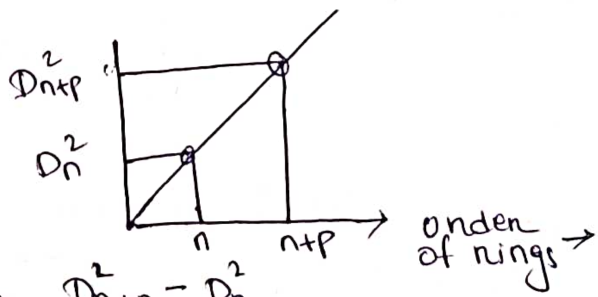
$$D_n = 2r_n = 2\sqrt{n\lambda R} \quad \{\text{for air } \mu = 1\}$$

$$D_n^2 = 4n\lambda R$$

$$D_{n+p}^2 = 4(n+p)\lambda R$$

$$D_{n+p}^2 - D_n^2 = 4p\lambda R$$

$$\boxed{\lambda = \frac{D_{n+p}^2 - D_n^2}{4pR}}$$



$$\text{slope} = \frac{D_{n+p}^2 - D_n^2}{n+p-n}$$

$$\text{slope} = \frac{D_{n+p}^2 - D_n^2}{p}$$

$$\therefore \boxed{\lambda = \frac{\text{slope}}{4R}}$$

a)

Interference	Diffraction
i) Interference fringes are formed when 2 light rays from the conerent sources are superimposed	i) Fringes are formed due to super-position of light rays from the same source.
ii) All bright fringes are equally illuminated	ii) Central fringe is brightest & all other fringes have intensity in decreasing order
iii) Width of all fringes are always equal	iii) Fringe width decreases when we move away from the centre.
iv) Minimum intensity is zero, so. fringes can be distinguished very well.	iv) Minimum intensity fringes are not perfectly darker so it is relatively difficult to distinguish bright & dark fringes.

b) slit width $= a = 18 \times 10^{-5}$ cm $= 18 \times 10^{-7}$ m

wavelength $= \lambda = 9000$ Å $= 9000 \times 10^{-10}$ m

$$a \sin\theta = n\lambda$$

$$\sin\theta = \frac{\lambda}{a} \quad (n=1)$$

$$\theta = \sin^{-1}\left(\frac{\lambda}{a}\right) = \sin^{-1}\left(\frac{9000 \times 10^{-10}}{18 \times 10^{-7}}\right)$$

$$= \sin^{-1}\left(\frac{1}{2}\right)$$

$\Rightarrow \boxed{\theta = 30^\circ}$ → half angular width of central bright band.

Q3.)

b) Mass of earth = ~~3f~~ 6×10^{24} kg = m_{earth}

Velocity of earth = 30 km/s = 3×10^{4} m/s = V_{earth}.

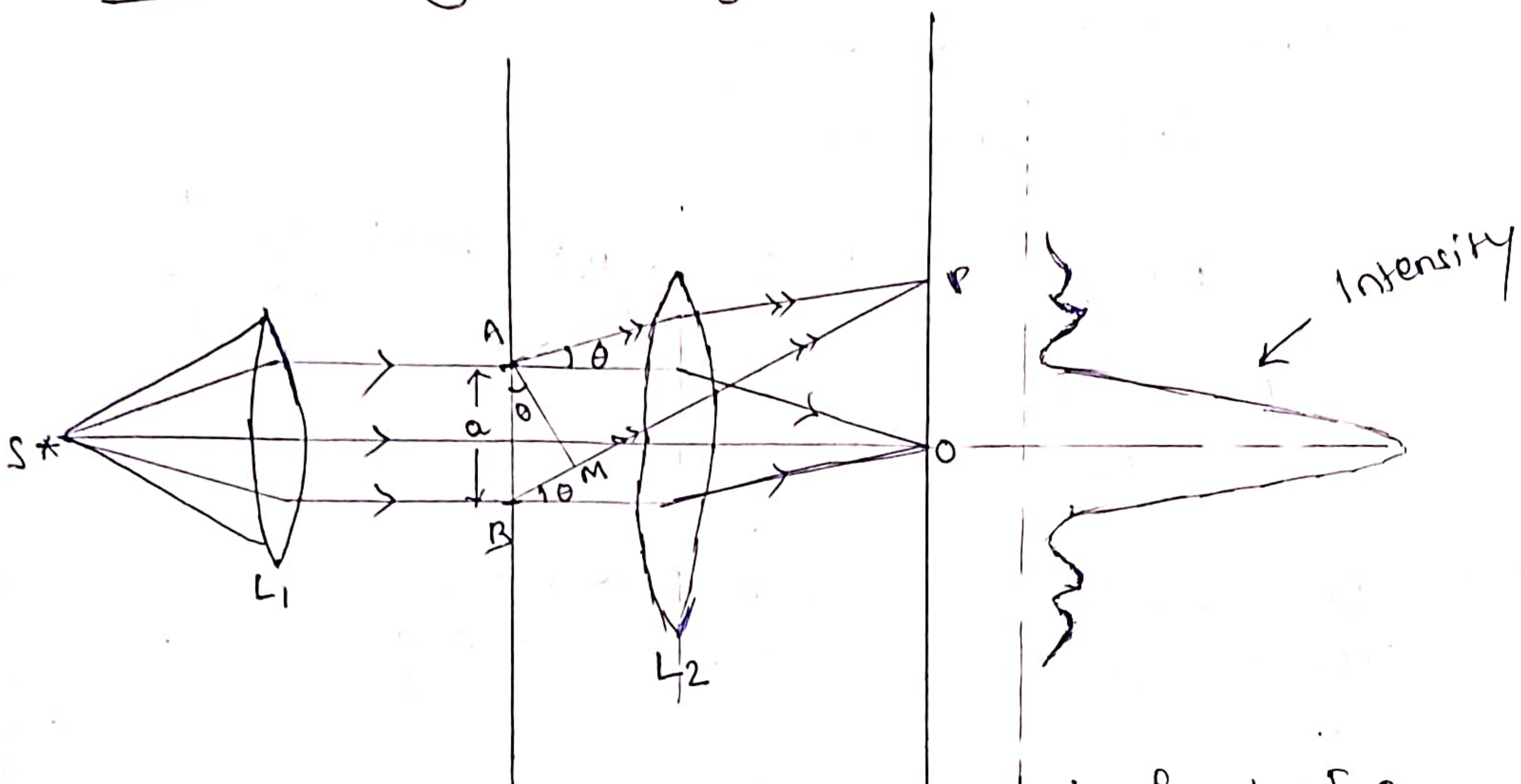
De-Broglie wavelength, $\lambda = \frac{h}{p} = \frac{h}{m_{earth} V_{earth}}$

$$\lambda = \frac{6.63 \times 10^{-34}}{6 \times 10^{24} \times 3 \times 10^{4}} \text{ m}$$

$$\boxed{\lambda = 3.6811 \times 10^{-63} \text{ m}}$$

We can't experience the wave nature of earth because of its very small wavelength which is of order 10^{-63} m.

Fraunhofer Single Slit Diffraction



Let a slit AB of width 'a' be placed in front of a source 'S'. A beam of monochromatic light of wavelength 'λ' from the source is incident on it through lens L_1. The diffracted beam from the slit is foccussed on the screen with the ~~to~~ help of lens L_2.

A diffraction pattern having alternate dark & bright fringes of decreasing intensity is obtained on the screen. on both sides of the central point 'O'.

As a plane wavefront is incident on the slit AB, each point of the slit becomes a source of secondary waves. The secondary wavelets travelling in a direction paraller to 'SO' & come to focus ~~on~~ at the point O.

Since the path travelled by the beams from upper & lower parts are same. Therefore, point O has maximum intensity.

Suppose some of the secondary waves are travelling in a direction making an angle θ with 'SO' & let focus at point 'P'.

Now, the path difference between the rays reaching at point P from upper & lower parts of slit is:

$$\Delta = BM = a \sin\theta .$$

$$\text{Phase difference} = \frac{2\pi}{\lambda} \cdot a \sin\theta$$

Suppose, the whole slit AB is divided into n equal parts. Therefore, the phase difference between the rays coming from 2 consequitive parts is given by:

~~Phase diff,~~ $$\boxed{\delta = \frac{1}{n}\left\{\frac{2\pi}{\lambda} a \sin\theta\right\}}$$

According to formula of resultant of n simple harmonic waves of equal amplitude 'a' & phase difference increasing by a constant value of 'δ'.

We can write the resultant amplitude:

$$R = \frac{a \sin n\delta/2}{\sin \delta/2} \Rightarrow R = \frac{a \sin \frac{n}{2}\left\{\frac{1}{n}\frac{2\pi}{\lambda} \cdot a\sin\theta\right\}}{\sin\left\{\frac{1}{2n}\frac{2\pi}{\lambda} \cdot a\sin\theta\right\}}$$

$$\Rightarrow R = \frac{a \sin\left\{\frac{\pi}{\lambda} a \sin\theta\right\}}{\sin\left\{\frac{\pi a \sin\theta}{n\lambda}\right\}}$$

$$\Rightarrow R = a\frac{\sin\alpha}{\sin \alpha/n} \qquad \left(\text{let } \alpha = \frac{\pi}{\lambda} a \sin\theta\right)$$

Since n is very large.

$$\Rightarrow R = a\frac{\sin\alpha}{\alpha/n} = na\frac{\sin\alpha}{\alpha}$$

$$\boxed{R = A\frac{\sin\alpha}{\alpha}} \qquad (A = na)$$

Now, Intensity, $I = R^2$

$$I = A^2 \sin^2\alpha/\alpha^2$$

$$\boxed{I = I_0 \frac{\sin^2\alpha}{\alpha^2}} \quad \text{---} (1) \qquad (I_0 = A^2)$$

Now, eqn ① represents the resultant intensity at the screen due to a single slit.

Position of principal maxima:

The resultant amplitude is given by:

$$R = \frac{A \sin\alpha}{\alpha}$$

$R \to R_{max}$ when $\alpha \to 0$

$$R_{max} = \lim_{\alpha \to 0} \frac{A \sin\alpha}{\alpha} = A \lim_{\alpha \to 0} \underbrace{\frac{\sin\alpha}{\alpha}}_{1} = A$$

$$\boxed{R_{max} = A}$$

$$I_{max} = R_{max}^2 = A^2 = I_0$$

Position: $\alpha \to 0$

$$\Rightarrow \frac{\pi}{\lambda} a \sin\theta \to 0$$

$$\Rightarrow \sin\theta \to 0$$

$$\Rightarrow \boxed{\theta \approx 0^\circ}$$

Position of minima:

The resultant amplitude is given by:

$$R = \frac{A \sin\alpha}{\alpha}$$

$R \to R_{min}$ when $\sin\alpha = 0$

Again $\alpha \neq 0$

$$\Rightarrow \sin\alpha = \sin \pm n\pi$$

$$\Rightarrow \boxed{\alpha = \pm n\pi}$$

$$\frac{\cancel{\pi}}{\lambda} a \sin\theta = \pm n \cancel{\pi}$$

$$\Rightarrow \boxed{a \sin\theta = \pm n\lambda} \quad (n = 1, 2, 3 \ldots)$$

$$\sin\theta = \pm \frac{n\lambda}{a} \quad (n = 1, 2, 3 \ldots)$$

$$\sin\theta = \pm \frac{n\lambda}{a}$$

$$\underbrace{\sin\theta_1 = \pm \lambda/a \;;\; \sin\theta_2}_{\lambda/a} = \pm \underbrace{\frac{2\lambda}{a} \;;\; \sin\theta_3}_{\lambda/a} = \pm \frac{3\lambda}{a}$$

Minima are equispaced

Position of secondary maxima

In addition to the principal maxima, there are less intense secondary maxima on both sides of the principal maxima. Now, the position of secondary maxima are given by:

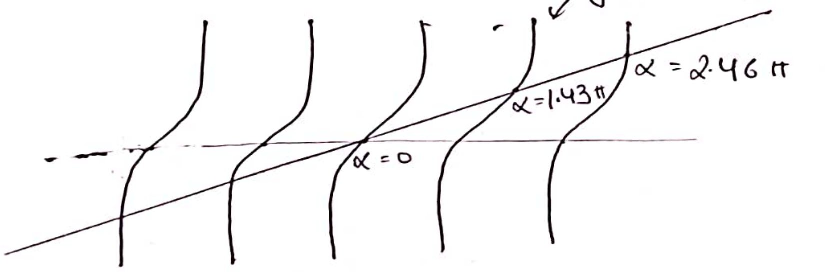
$$\frac{dI}{d\alpha} = 0 \Rightarrow \frac{d}{d\alpha}\left(A\,\frac{\sin^2\alpha}{\alpha^2}\right) = 0$$

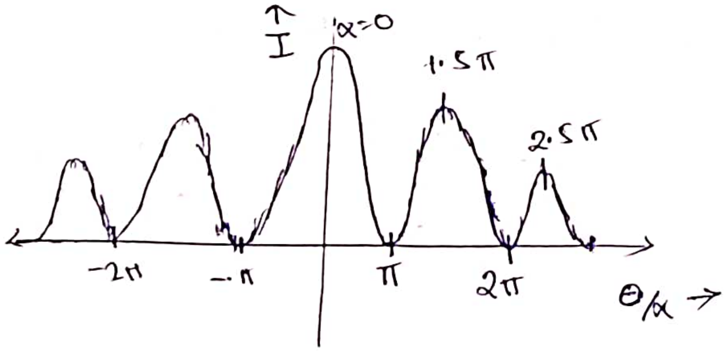
$$\Rightarrow A\,\frac{2\sin\alpha}{\alpha}\left(\frac{\alpha\cos\alpha - \sin\alpha}{\alpha^2}\right) = 0$$

$$\Rightarrow \sin\alpha = 0 \quad \text{or} \quad \alpha = \tan\alpha$$

Since, $\sin\alpha = 0$ gives position of principal maxima, Thus, the position of secondary maxima can be obtained from the root of the eqn:

$\alpha = \tan\alpha$:





from the graph α v/s $\tan\alpha$, the point of intersection is obtained at $\alpha = 1.43\pi$ & 2.46π & so on.. which gives the position of secondary maxima.

Now, taking $\alpha = 0, \frac{3\pi}{2}, \frac{5\pi}{2}$ ~~also~~ & so on we can calculate the intensity of principal maxima, 1'st secondary maxima, 2'nd secondary maxima. & so on.

i) For the principal maxima:

$$\theta = 0 \; ; \; \alpha \to 0$$

$$I = I_0 \frac{\sin^2\alpha}{\alpha^2} \Rightarrow \boxed{I = I_0}$$

ii) For ~~2~~ 1'st secondary maxima:

$$\alpha = \frac{3\pi}{2}$$

$$I_1 = I_0 \frac{\sin^2 \frac{3\pi}{2}}{(3\pi/2)^2} = I_0 \frac{1}{\frac{9\pi^2}{4}} = \frac{I_0}{22}$$

iii) For 2'nd secondary maxima:

$$\alpha = 5\pi/2$$

$$I_2 = I_0 \frac{\sin^2 5\pi/2}{(5\pi/2)^2} = \frac{I_0}{\frac{25\pi^2}{4}} = \frac{I_0}{61}$$

iv) $I_3 = \frac{I_0}{121}$

Intensity ratio:

$$I_0 : I_1 : I_2 : I_3 : \dots = 1 : \frac{4}{9\pi^2} : \frac{4}{25\pi^2} : \frac{4}{49\pi^2} \dots$$

$$= 1 : \frac{1}{22} : \frac{1}{61} : \frac{1}{121} \dots$$

$$= 1 : 0.045 : 0.0164 : 0.008$$

This indicates that the intensity of principal maxima is the highest & the intensity ~~decreases~~ of secondary maxima decreases on either side of I_0 with increase of order number.

Uncentainity Principle:

The principle states that for a particle of atomic magnitude in motion, it is impossible to determine both the position & momentum simultaneously with perfect accuracy.

① Quantitatively the principle is represented by Hisenberg's uncentainity relation which is:

"The product of the uncentainity 'Δx' (possible error) in the x-coordinate of the particle at some instant & the uncentainity Δp_x in the x coordinate of the momentum at the same instant is of the order or greater than $\hbar$.

$$\left.\begin{aligned} \Delta x \cdot \Delta p_x &\gtrsim \hbar \\ \Delta y \cdot \Delta p_y &\gtrsim \hbar \\ \Delta z \cdot \Delta p_z &\gtrsim \hbar \end{aligned}\right\} \quad \text{—} (1)$$

$$\hbar = \frac{h}{2\pi}$$

② 2 more canonical conjugates whose uncentainity relations:

$$\Delta E \cdot \Delta t \gtrsim \hbar \quad \text{—} (2)$$

$$\Delta L_z \cdot \Delta \phi \gtrsim \hbar \quad \text{—} (3)$$

③ Minimum value of product of uncentainity:

$$\Delta x \cdot \Delta p_x = \frac{\hbar}{2}$$

④ While solving numerical: (if min. not mentioned)

$$\Delta x \cdot \Delta p = \hbar$$

Proof for e^- can't be present inside a nucleus:

Let e^- exist inside a nucleus then the uncertainity in the position of the e^- will be same as the diameter of the nucleus.

$$\Delta x = 2\times10^{-14}\,m.$$

Now, the minimum uncertainity ΔP_x is: $\boxed{\hbar = \frac{h}{2\pi}}$

$$\Delta P_x = \frac{\hbar}{\Delta x} = \frac{6.23\times10^{-34}}{2\times3.14\times2\times10^{-14}}$$

$$= 5.278\times10^{-21}\ kg\ m/sec$$

If e^- exist inside the nucleus then its minimum momentum:

$$P_{min} = 5.278\times10^{-21}\ kg\text{-}m/sec.$$

Now, for electron:

$$E^2_{min} = P^2_{min}c^2 + m_0^2c^4 = [5.278\times10^{-21}\times3\times10^8]^2 + [9.11\times10^{-31}]^2\cdot[3\times10^8]^4$$

$$= [2.5\times10^{-24} + \underbrace{6.72\times10^{-27}}_{\approx 0}]\ J^2$$

$$E^2_{min} = 2.5\times10^{-24}\ J^2$$

$$\Rightarrow E_{min} = \sqrt{2.5\times10^{-24}} = 1.58\times10^{-12}\ J$$

$$\Rightarrow E_{min} = \frac{1.58\times10^{-12}}{1.6\times10^{-19}}\ eV = 9.875\times10^{6}\ eV$$

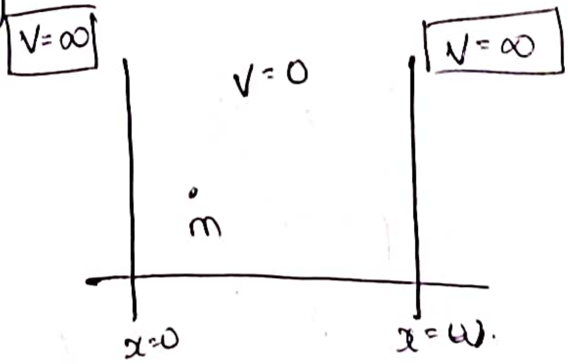
$$E_{min} = 9.8\,MeV \approx 10\ MeV.$$

Hence, for the e^- to be present inside the nucleus its min energy should be of the order of 10MeV.

But the experiment shows that the maximum energy of β particle emitted heavy radioactive nuclei is ~~4eV~~ 4MeV.

∴ e^- can't exist within a nucleus.

Particle in 1D box of infinite height



Suppose a particle of mass 'm' bouncing back and forth between the wall of a 1D box.
Considering that the particle moves in the x-axis from $x=0$ to $x=\omega$ ~~with~~ & the walls of the box are infinitely hard.
We consider that the particle does n't lose energy when it collides with the walls ∴ So, that its total energy remains constant.
The box can be represented as by a potential well or box of width 'ω', so that the potential energy 'V' of the particle is infitely ~~f~~ high on both sides of the box. while on inside, throughout the length 'ω', V is uniform (0).

$$V(x) = 0 \quad \text{for} \quad 0 < x < \omega$$
$$= \infty \quad \text{for} \quad x > \omega \text{ or } x < 0$$

Schrodinger time independent eqn:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2}(E - V)\psi = 0$$

Here $V = 0$:

$$\frac{\partial^2 \psi}{\partial x^2} + \frac{2m}{\hbar^2} E \psi = 0$$

$$\frac{d^2\psi}{dx^2} + k^2\psi = 0 \quad \text{---} (1) \qquad k = \sqrt{\frac{2mE}{\hbar^2}}$$

Soln. of eqn (1):

$$\psi = A\sin kx + B\cos kx \quad \text{---} (2)$$

where A & B are arbitrary constants that can be found by applying the boundary conditions.

Boundary condition-1:

$x = 0;\ \psi = 0$

$0 = A\sin 0 + B\cos 0 \Rightarrow B = 0$

$$\Rightarrow \boxed{\psi = A\sin kx} \quad \text{---} (3)$$

Boundary condition-2:

$\cancel{x=0;}$ $x = \omega;\ \psi = 0$

$0 = A\sin k\omega$ --- (4)

Now A can't be 0 because if $A=0$, then there would be no soln. $\Rightarrow A \neq 0$.

$\Rightarrow$ Eqn (4) valid if $\sin kx = 0$

$\sin k\omega = \sin n\pi$

$\Rightarrow k\omega = n\pi$ where $n = 1, 2, 3 \ldots$

$$\Rightarrow \boxed{k = \frac{n\pi}{\omega}} \quad n = 1, 2, 3 \ldots$$

This is a necessary condn for the soln of wave function to exist.

We can't take $n=0$ ~~~~ because if $n=0 \Rightarrow k=0 \Rightarrow E=0$

Hence ~~for~~ $\psi(x)=0$ everywhere in box.

This means a particle with zero energy can't be present inside the box.

This means particle can't have zero energy.

Hence the wave function for motion of particle in region $0<x<a$ is given by:

$$\boxed{\psi = A\sin \frac{n\pi}{\omega} x} \quad \text{---} (5)$$

Energy eigen value:

$$K = \frac{n\pi}{\omega}$$

Also, $K = \sqrt{\frac{2mE}{\hbar^2}} \Rightarrow K^2 = \frac{2mE}{\hbar^2} = \frac{n^2\pi^2}{\omega^2}$

$$\Rightarrow E = \frac{n^2\pi^2\hbar^2}{2m\omega^2} \quad \left(\hbar = \frac{h}{2\pi},\ \hbar^2 = \frac{h^2}{4\pi^2}\right)$$

$$\Rightarrow E = \frac{n^2 \cancel{\pi^2} \cdot \frac{h^2}{4\cancel{\pi^2}}}{2m\omega^2} \Rightarrow \boxed{E_n = \frac{n^2h^2}{8m\omega^2}} \quad (n = 1, 2, 3 \ldots)$$

for $n=1$ (zeropoint energy):

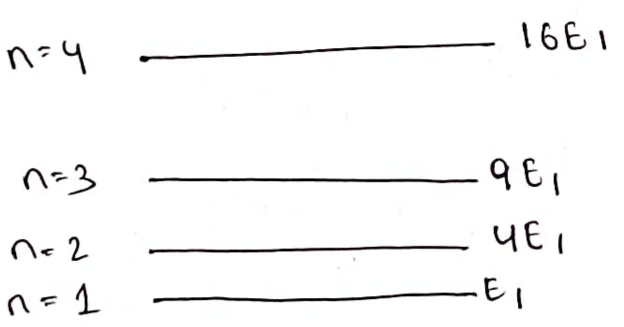
$$\boxed{E_1 = \frac{h^2}{8m\omega^2}}$$

* $E_n = n^2 E_1$ $(n = 1, 2, 3 \ldots)$ → Energy changes are discreete in nature.
* $E_n = n^2 E_1$

$E_{n+1} = (n+1)^2 E_1 \Rightarrow E_{n+1} - E_n = \{(n+1)^2 - n^2\} E_1$
$= (2n+1) E_1$

$\Rightarrow$ $\boxed{\text{Energy spacing} \propto 2n+1}$

$\Rightarrow$ Energy levels are not ~~equa~~ equispaced.



Proof: Momentum are equispaced:

$$E_n = \frac{n^2h^2}{8\cancel{m}\omega^2} = \frac{P_n^2}{2\cancel{m}}$$

$$P_n^2 = \frac{n^2h^2}{4\omega^2} \Rightarrow P_n = \frac{nh}{4\omega}$$

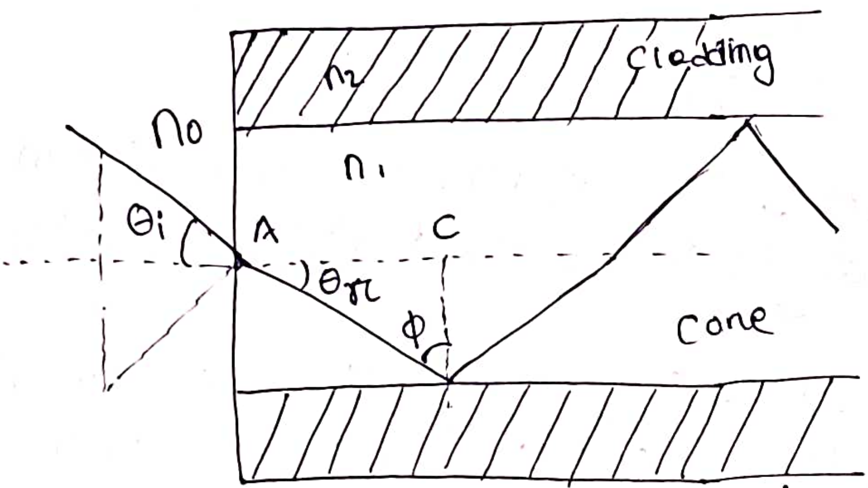
$P_1 = \frac{h}{4\omega}$

$P_2 = \frac{2h}{4\omega}$] $h/4\omega$

$P_3 = \frac{3h}{4\omega}$] $h/4\omega$

$\Rightarrow$ Momenta are equispaced.

Let n_1 is refractive index of core
$n_2 \rightarrow$ refractive index of cladding.
$(n_2 < n_1)$
$n_0 \rightarrow$ refractive index of medium from which the light ray is incident.



Let a ray enter at an angle 'θ_i' from axis & refract at core at angle 'θ_r'.
The ray strikes the core-cladding interface at an angle 'ϕ'.
The rays with angle greater than critical angle ϕ_c, at the core-cladding interface will be transmitted by Total internal Reflection:
By Snell's law:

$$n_0 \sin\theta_i = n_1 \sin\theta_r .$$

$$\& \quad \theta_r + \phi = 90^\circ \Rightarrow \theta_r = 90^\circ - \phi$$

$$n_0 \sin\theta_i = n_1 \sin(90^\circ - \phi) = n_1 \cos\phi$$

when $\phi = \phi_c$, then $\theta_i = \theta_{imax}$.

$$\text{So, } \sin\theta_{imax} = \frac{n_1}{n_0} \cos\phi \quad \text{—(1)}$$

Now using law of Total Internal Reflection:

$$\sin\phi_c = \frac{n_2}{n_1}$$

$$\therefore \cos\phi_c = \frac{\sqrt{n_1^2 - n_2^2}}{n_1} \quad \text{—(2)}.$$

Putting (2) in (1):

$$\sin \theta_{imax} = \frac{A_1}{n_0} \cdot \frac{\sqrt{n_1^2 - n_2^2}}{A_1} = \frac{\sqrt{n_1^2 - n_2^2}}{n_0}$$

for air $n_0 = 1$:

$$\sin \theta_{imax} = \sqrt{n_1^2 - n_2^2}$$

let $\theta_{imax} = \theta_G$

then $\theta_G = \sin^{-1}\left(\sqrt{n_1^2 - n_2^2}\right)$

$\theta_G \rightarrow$ Acceptance Angle. It is defined as the maximum external incidence angle for which light will propagate in the fiber.

It is also defined as the half angle of the cone with which the light is totally ~~in~~ reflected by the fiber cone.

The light gathering ability of ~~bi~~ fiber is related to its acceptance angle & expressed as numerical ~~ano~~ aperature of the fiber.

$$NA = \sin \theta_G = \sqrt{\frac{n_1^2 - n_2^2}{n_0^2}}$$

for air: $n_0 = 1$:

$$NA = \sin \theta_G = \sqrt{n_1^2 - n_2^2}$$

~~Numerical Aperature:~~

Numerical Aperature:

NA of a optical fiber is defined as the sine of the acceptance angle & measures the accepting power of the fiber.

So, $NA = \sin\theta_a = \sqrt{n_1^2 - n_2^2}$

$$n_1^2 - n_2^2 = (n_1+n_2)(n_1-n_2)$$
$$= \left(\frac{n_1+n_2}{2}\right)\left(\frac{n_1-n_2}{n_1}\right) \cdot 2n_1$$

Putting $\frac{n_1+n_2}{2} \approx n_1$ & $\frac{n_1-n_2}{n_1} = \Delta$: fractional change of Refractive Index

$$n_1^2 - n_2^2 = 2n_1^2\Delta$$

$$\therefore \boxed{NA = n_1\sqrt{2\Delta}}$$

It provides a measure of light gathering capacity of the fiber. (0.13 → 0.50) range.

Larger the value of NA → larger is the energy gathered by fiber from source.

NA depends on:

RI of core 'n_1'

" " clading 'n_2'

Applications:

→ Use in communication (becoz of large band width)

→ transmission of digital data.

→ application in security & alarm systems, industrial automation, etc.

→ use in defense → becoz → high privacy

→ command & control link on ship & aircraft, data links for satellite earth stations.

Electromagnetic Wave eqn / Classical Wave Eqn:

Maxwell's electromagnetic eqn lead to the wave eqn for electric & magnetic fields in vaccum:

$$\vec{D} = \varepsilon_0 \vec{E} \quad \& \quad \vec{H} = \vec{B}/\mu_0$$

So, maxwell eqns in absence of changes & current $(\rho = 0 ; \vec{J} = 0)$ in vaccum become:

$$(1) \quad \vec{\nabla} \cdot \vec{E} = 0$$

$$(2) \quad \vec{\nabla} \cdot \vec{B} = 0$$

$$(3) \quad \vec{\nabla} \times \vec{E} + \frac{\partial \vec{B}}{\partial t} = 0$$

$$(4) \quad \vec{\nabla} \times \vec{B} - \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} = 0$$

Taking curl of eqn (3):

$$\vec{\nabla} \times \left\{ (\vec{\nabla} \times \vec{E}) + \frac{\partial \vec{B}}{\partial t} \right\} = 0 \Rightarrow \vec{\nabla} \times (\vec{\nabla} \times \vec{E}) + \vec{\nabla} \times \frac{\partial \vec{B}}{\partial t} = 0$$

$$\vec{\nabla} \times (\vec{\nabla} \times \vec{E}) = \vec{\nabla} (\underbrace{\cancel{\vec{\nabla} \cdot \vec{E}}}_{0}) - \nabla^2 \vec{E} \quad (\text{using } (1))$$

$$\Rightarrow -\nabla^2 \vec{E} + \frac{\partial}{\partial t} (\vec{\nabla} \times \vec{B}) = 0 \quad \text{— (4)}$$

Maxwell's 4th Rel^n say:

$$\vec{\nabla} \times \vec{B} = \mu_0 \varepsilon_0 \frac{\partial \vec{E}}{\partial t} \rightarrow \text{putting in } (4)$$

$$\boxed{\nabla^2 \vec{E} - \mu_0 \varepsilon_0 \frac{\partial^2 \vec{E}}{\partial t^2} = 0}$$

Classical wave

$$\Rightarrow \boxed{\nabla^2 \vec{E} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0} \rightarrow \text{For 3D}$$

$$\Rightarrow \boxed{\frac{\partial^2 E_x}{\partial x^2} - \frac{1}{c^2} \frac{\partial^2 \vec{E}}{\partial t^2} = 0} \rightarrow \text{For 1-D}$$

Classical wave Eqn