Assignment-4

PHY 323/PHY 3230/MSP 323

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1. Recall the expression of the electric and magnetic field due to a charge particle moving in an arbitrary trajectory from the exercise given in the lecture note.

   (a) Show that the Poynting vector is

   $$S = \frac{c}{4\pi}[E^2\hat{\mathbf{R}} - (\hat{\mathbf{R}}.\mathbf{E})\mathbf{E}]$$

   where $\hat{\mathbf{R}}$ is the unit vector along the vector connecting the observation point and the retarded position of the charge.

   (b) Now suppose at retarded time t_r, the instantaneous velocity of the charge is zero. Show that the Poynting vector is

   $$S = \frac{cq^2a^2}{4\pi R^2}\sin^2\theta$$

   where $\mathbf{a}$ is the acceleration of the charge and θ is the angle between $\hat{\mathbf{R}}$ and $\mathbf{a}$.

   (c) Find the power radiated in the direction $\hat{\mathbf{R}}$ i.e., $\frac{dP}{d\Omega}$. Find the total power radiated.

   (d) Now suppose the particle is moves as

   $$z(t) = a\cos(\omega t)$$

   Find time averaged $\frac{dP}{d\Omega}$ for this case and plot θ vs $\frac{dP}{d\Omega}$.

2. Recall that the field strength that we obtain from the Lienard Wiechart potential is:

   $$F_{\mu\nu} = -\frac{q}{r}\left[2k_{[\mu}a_{\nu]} + 2k\cdot ak_{[\mu}u_{\nu]}\right] + \frac{2q}{r^2}k_{[\mu}u_{\nu]}$$

   Check the above result by explicitly showing that it satisfies the source-free Maxwell's equation $\partial_\mu F^{\mu\nu} = 0$ away from the charge.

3. Consider a charged particle of charge q moving along a circle of radius a with a uniform speed v. Assume $v << c$. Obtain the electric field $\vec{E}$ and and magnetic field $\vec{B}$ at a point in the wave zone. Find the time average of the angular distribution $\langle\frac{dP}{d\Omega}\rangle$ and the total power $\langle P\rangle$ radiated by the system.

4. Consider two charged particles of the same charge q moving along a circle of radius a with a uniform speed v such that they are always diametrically opposite to each other. Obtain the electric field $\vec{E}$ and and magnetic field $\vec{B}$ at a point in the wave zone. Find the time average of the angular distribution and the total power radiated by the system.

5. Picture two tiny metal spheres separated by a distance d and connected by a fine wire. At time t the charge on the upper sphere is $q(t)$, and the charge on the lower sphere is $-q(t)$. Suppose that we drive the charge back and forth through the wire, from one end to the other, at an angular frequency ω (Figure below)

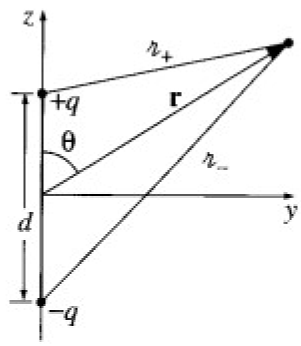

Figure 1: Caption

(a) Using the dipole approximation calculate the Electric field and magnetic field at some point in the wave zone. And then calculate Poynting vector.

(b) What is the time average of the angular distribution and the total power radiated by this dipole **in SI units**?

(c) Find the radiation resistance of the wire joining the two ends of the dipole. (This is the resistance that would give the same average power loss to heat as the oscillating dipole in fact puts out in the form of radiation.) Show that$R = 790(d/\lambda)^2$ Ohm, where λ is the wavelength of the radiation (Use $\epsilon_0 = 8.854 \times 10^{-12} Fm^{-1}$, $c = 3 \times 10^8 ms^{-1}$). For the wires in an ordinary radio (say, d = 5 cm), should you worry about the radiative contribution to the total resistance?

6. There is a magnetic field given as below:

$$\vec{B} = B_0\hat{k} \text{ , when } x \geq 0$$
$$= 0 \text{ , when } x < 0$$

A particle of mass m and charge q moving with a uniform velocity $\vec{v} = v_0\hat{i}$ enters the magnetic field zone ($x \geq 0$) from the non magnetic field zone ($x < 0$) at $t = 0$. Due to the bending by the magnetic field, it comes out of the magnetic field zone after some time.

(a) How much total energy E_D is radiated by the particle as dipole radiation?

(b) How much total energy E_Q is radiated by the particle as quadrupole radiation? What is the ratio $\frac{E_Q}{E_D}$?

(c) Is there any magnetic dipole radiation? Give appropriate justification.