Problem Set 3 – due at 5:00 PM on Friday, April 21

1. Suppose that the demand side of a competitive labor market consists of 100 medium-sized employers, each with daily labor demand curve $E^d = 200 - w$, and 10 large employers, each with daily labor demand curve $E^d = 2000 - 10w$, where E^d is the number of workers demanded per day when the daily wage per worker is w. Suppose that the supply side of this labor market consists of 20,000 potential workers, whose reservation values for total daily compensation are distributed uniformly over the interval [100,200]. At any compensation level above the reservation value, each worker supplies one unit of labor per day; at any compensation level below the reservation value, each worker supplies zero units of labor to the labor market.

    a. Find the market labor demand curve and the market labor supply curve.
    b. Find the competitive labor market equilibrium. How many workers are employed in each type of firm?
    c. In an effort to improve public health, the government requires that all employers provide health club memberships to all of their employees. Suppose that the cost of providing this benefit for each firm is \$10 per day per worker and that the value of this benefit to each worker is \$6 per day. Find the new labor market equilibrium (i.e., total employment, the daily money wage per worker, the daily per worker cost of employment to firms, and the daily value of employment to workers). Again, how many workers are employed in each type of firm?
    d. Repeat part (c), now making the additional assumption that large firms can purchase health club memberships in bulk, thereby lowering large firms' cost of providing this benefit to \$6 per worker per day.
    e. Repeat part (d), now making the further additional assumptions that: 50% of (potential) workers value health club membership at \$6 per day; 50% of (potential) workers value health club membership at only \$2 per day; and a worker's valuation of health club benefits is independent of the worker's reservation value of total daily compensation.

2. Suppose that pharmaceutical sales representative i derives utility from his/her job according to the utility function $U_i(W,T) = W - \beta(T - T_i^*)^2$, where W denotes annual salary, $T \in [0,1]$ is the fraction of annual work days spent away from home, $\beta > 0$ is a taste parameter, and T_i^* is sales representative i's preferred fraction of annual work days spent travelling. Suppose that sales reps have heterogeneous preferences for on-the-job travel; in particular, suppose that T_i^* is uniformly distributed on the $[0,1]$ interval in the sales rep population.

   On the demand side of the pharmaceutical sales rep labor market, suppose that a fixed fraction p of jobs require "low travel", $T = T_L$, and a fixed fraction $1 - p$ of jobs require "high travel", $T = T_H$, where $0 \leq T_L < T_H \leq 1$.

   a. Illustrate the typical sales rep's indifference curves in the T-W plane.
   b. Explain in words what condition must hold if a particular pair of salaries, (W_L, W_H) — one for "low travel" jobs and one for "high travel" jobs — is an equilibrium.
   c. Express the condition in (b) as an equation. (Hint: Given worker preferences, which sales representative must be indifferent between the two jobs?)
   d. Manipulate the equation in (c) to obtain an expression for the equilibrium salary differential between "high travel" and "low travel" jobs, $W_H - W_L$, as a function of the parameters $\{\beta, p, T_L, T_H\}$. Is the equilibrium compensating differential unambiguously signed?
   e. Determine how a ceteris paribus change in each of the parameters affects the equilibrium compensating differential, and interpret your results.

3. Consider an agent who works for two periods and has the option to make a human capital investment in the first period. If the agent does **not** invest in human capital in period 1, her earnings profile will be flat: $Y_1 = Y_2 = Y$, where Y_t is the agent's earnings in period t. If the agent **does** invest in period 1, she forgoes some earnings in period 1 but reaps a return in period 2; in this case, her earnings profile is $Y_1 = (1-\theta)Y$, $Y_2 = (1+\pi)Y$, where $0 < \theta < 1$ and $\pi > 0$.

   The agent's lifetime utility is given by $U(C_1, C_2) = \ln C_1 + \frac{1}{1+\rho} \cdot \ln C_2$, where C_t is the agent's consumption in period t and $\rho \geq 0$ is the agent's personal discount rate (i.e., the rate at which period 2 utility is discounted relative to period 1 utility). The agent's full utility maximization problem is to make the optimal investment decision (which is a binary choice — invest or don't invest) **and** to make the optimal consumption allocation across periods among all feasible allocations. This problem asks you to analyze how the presence or absence of a loan market to finance investments in human capital affects these decisions (and therefore does **not** assume at the outset the agent's goal is to maximize the present discounted value of **earnings**).

   a. Suppose that the agent can borrow (or save) at interest rate $r > 0$. In this case the agent's budget constraint is $C_2 = Y_2 + (1+r)(Y_1 - C_1)$, where first period consumption is constrained to the interval, $0 \leq C_1 \leq Y_1 + \frac{Y_2}{1+r}$.

      i. What consumption bundle will the agent choose if she does **not** invest in human capital and what is her utility in this case?
      ii. What consumption bundle will the agent choose if she **does** invest in human capital and what is her utility in this case?
      iii. Find and interpret a condition that determines whether or not the agent invests in human capital.

   b. Now suppose that the agent **cannot borrow at all** (but still can save at interest rate r). The inability to borrow changes the agent's budget constraint to $C_2 = Y_2 + (1+r)(Y_1 - C_1)$, but with first period consumption constrained to the narrower interval, $0 \leq C_1 \leq Y_1$.

      i. What consumption bundle will the agent choose if she does **not** invest in human capital and what is her utility in this case?
      ii. What consumption bundle will the agent choose if she **does** invest in human capital and what is her utility in this case? (Hint: Assume that the agent's value of ρ is high enough that she chooses zero savings in the first period if she invests and therefore chooses the consumption allocation: $C_1^* = (1-\theta)Y$ and $C_2^* = (1+\pi)Y$.)
      iii. Find and interpret a condition that determines whether or not the agent invests in human capital. Interpret the difference between this condition and the one that you found in part (a).

4. Consider a competitive labor market with two types of labor, A and B, and many employers with varying intensity of prejudice against type B workers. In particular, there are a total of 100 employers, each with a "discrimination coefficient" of $d_i \in \{0,1,2,3,4\}$, and with exactly 20 employers of each "prejudice intensity".

   The value of the marginal product of labor function is identical for all of the employers: $VMP_i = 20 - 0.5(E_{Ai} + E_{Bi})$, where E_{ji} is the quantity of labor type j employed by employer i. Note that this value of marginal product of labor function implies that the two labor types are perfect substitutes and equally productive in the production process.

   On the supply side of the labor market, there are 930 workers of type A and 310 workers of type B, each of whom supply one unit of labor perfectly inelastically (and therefore all of whom must be employed in the labor market equilibrium).

   Finally, let w_A and w_B denote the market wage of each labor type (which every individual agent takes as given since the labor market is competitive).

   a. Find the individual employer demand functions for type A and type B labor, as a function of the wage of each labor type, for each of the five employer prejudice types.
   b. Use your answer to (a), along with the number of employers of each type, to find the market demand functions type A and type B labor. (Hint: These are both multi-step functions, and so are slightly tedious to express algebraically.)
   c. Find the values of w_A and w_B in the competitive labor market equilibrium. (Hint: You need to find the wage values such that employers are willing to demand precisely the available quantities of each type of labor.)
   d. Which employers exclusively employ type A labor and how many type A workers do they employ? Which employers exclusively employ type B labor and how many type B workers do they employ? Do any employers employ both types of labor?
   e. Would the equilibrium wage differential between type A and type B workers change in response to a small change in the composition of the supply side of the labor market? Explain briefly.