Faculty of Engineering, Environment, and Computing

7065MAA Nonlinear Control Engineering

Assignment Brief 2022/23

Student Name:		SID:
Module Title: Nonlinear Control Engineering	Cohort: JANAPR	Module Code: 7065MAA
Coursework No./ Title: **7065MAA Coursework 2223**		Hand out date: 20/02/2023
Lecturer: Dr. Akin Delibasi		Hand in date: **03/04/2023 By 6:00 pm.**
Estimated time: 10h	Extension: applications must be made before **3** weeks	% of module marks: 10 credits out of 15 credits total = **66.67%**
Coursework type: problem based		Individual: ✓
		Group

Submission arrangements: **online.** A Turnitin (anti - plagiarism) submission link will be available via the **7065MAA** module assignment page on Aula. **Submit 1 file:** Coursework report in pdf/doc format.

The coursework document should contain the answers to all the questions together with comprehensive discussion and analysis, including all derivation steps as well as all relevant figures and documentation.

The file name of your pdf/doc should be in the following form: "7065MAA_CW_SID_StudentName"

Simulink plots should not be default Scope outputs (i.e. with black backgrounds and poor visibility). Plots should be clear, use sensible scaling, include axes labels with variable units, legend and have a title. An example of what is expected is shown below:

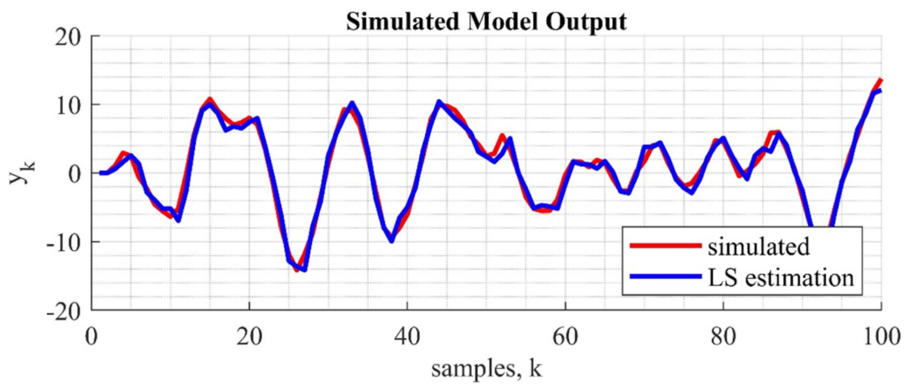

Figure 1.2, LS estimation and simulated output for given input

Submit your MATLAB/SIMULNK solution files via a separate HandIn submission link also available via the **7065MAA** module assignment page on Aula. **Submit 1 zipped file:**

Intended Learning Outcome for this Coursework:

1) Comprehensively recognise nonlinear characteristics that occur in practical processes;

2) Comprehensively understand the concept of equilibria in the context of nonlinear systems and know how to find the equilibria of given nonlinear dynamics;

3) Demonstrate systematic understanding of the knowledge in order to linearise nonlinear models and explain the limitations of the approach;

4) Critically evaluate and demonstrate the conceptual understanding to recall and apply time and frequency domain methodologies for stability analysis of nonlinear systems;

5) Perform controller design for various classes of nonlinear systems based on their model structures

6) Perform nonlinear control system analysis and design using simulation packages, e.g. MATLAB/Simulink

QUESTION 1 – Linearisation and Equilibrium points (25 marks)

1a) Explain the differences (at least 5 points) between linear and nonlinear systems and give an example for each point of differences.

(5 marks)

1b) Consider a system represented by a differential equation

$$\ddot{x}(t) + \dot{x}(t) + 4\,x(t) - x^3(t) = 0, \qquad t > 0.$$

i. Explain what is commonly understood by the "stability of a nonlinear system".

(2 marks)

ii. Express the system in state-space form $\dot{\bar{x}} = f(\bar{x})$, where $\bar{x} = [x_1 \quad x_2]^T = [x \quad \dot{x}]^T$. Then find the equilibrium points (equilibria) of the system.

(3 marks)

iii. Using the linearisation method, determine the stability of the origin. If it is stable, be precise with the type of stability that the origin possesses.

(5 marks)

1c) Of the equilibria obtained in (1b),

i. Find the type of each equilibrium and determine the stability of each of them.

(6 marks)

ii. Sketch the phase portrait of each equilibrium in one plot. (Note: please do not use MATLAB or any other computer aid to sketch it)

(4 marks)

Continue

QUESTION 2 – Lyapunov Stability Theory (30 marks)

Given a nonlinear system:

$$\dot{x}_1 = x_1 - 2\,x_1\,x_2 + u\,x_2$$
$$\dot{x}_2 = -x_2 + 2\,x_1^2$$

2a) Using

$$V(x_1, x_2) = \frac{1}{2}x_1^2 + \frac{1}{2}(x_2 - 1)^2, \qquad x_2 \geq 1$$

as a Lyapunov function for the system, determine the stability of the system for $u = 0$ and $x_2 \geq 1$.

(10 marks)

2b) Find all the equilibria of the system for $u = 0$, and determine the type and stability of each equilibrium.

(10marks)

2c) Relate your finding about the stability of the equilibria in (b) with the stability of the system in (a).

(10 marks)

Continue

QUESTION 3 – Nonlinear Controller Design (Backstepping) (15 marks)

Given a nonlinear system:

$$
\begin{aligned}
\dot{x}_1 &= -x_1^3 + x_2 \\
\dot{x}_2 &= x_3 \\
\dot{x}_3 &= u
\end{aligned}
$$

design a backstepping controller to stabilise the system.

Continue

QUESTION 4 – Nonlinear Controller Design (Feedback Linearisation) (30 marks)

Figure 1 shows a structure of a two-link planar manipulator system. The links are cascaded in a serial fashion and are actuated by individual motors.

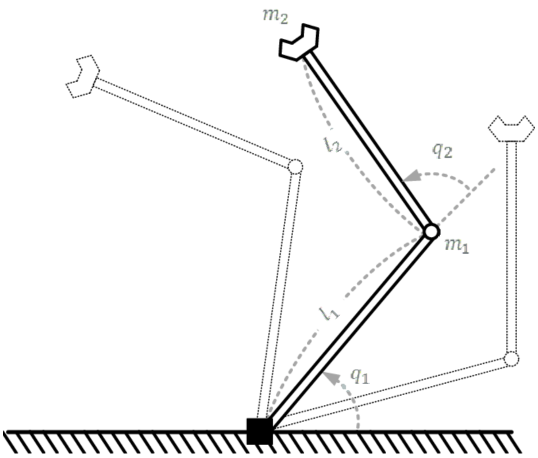

Figure 1 Two link planar robot manipulator (Baek, J. and Kwon, W., 2020)

Equation of motion of the two-link planar manipulator is described by

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + g(q) = \tau$$

where q and $\dot{q}$ are 2-dimensional vectors of generalized coordinates representing joint position and velocity respectively, $M(q)$ is a symmetric inertia matrix. The $C(q,\dot{q})\dot{q}$ accounts for centrifugal and Coriolis forces. $g(q)$ denotes gravity forces.

4a) A two-link planar robot manipulator model's system matrices are given as

$$M(q) = \begin{bmatrix} \theta_1 + 2\,\theta_3\cos(q_2) & \theta_2 + \theta_3\cos(q_2) \\ \theta_2 + \theta_3\cos(q_2) & \theta_2 \end{bmatrix},$$

$$C(q,\dot{q}) = \begin{bmatrix} -\theta_3\sin(q_2)\dot{q}_2 & -\theta_3\sin(q_2)\dot{q}_1 - \theta_3\sin(q_2)\dot{q}_2 \\ \theta_3\sin(q_2)\dot{q}_1 & 0 \end{bmatrix},$$

$$g(q) = \begin{bmatrix} \theta_4\,g\cos(q_1) + \theta_5\,g\cos(q_1+q_2) \\ \theta_5\,g\cos(q_1+q_2) \end{bmatrix},$$

where

$$\theta_1 = (m_1+m_2)l_1^2 + m_2 l_2^2,\ \ \theta_2 = m_2 l_2^2,\ \ \theta_3 = m_2 l_1 l_2,\ \ \theta_4 = (m_1+m_2)l_1,\ \ \theta_5 = m_2\,l_2.$$

Express the system in state-space form $\dot{x} = f(x) + u(x,\tau)$, where $x = [x_1 \quad x_2 \quad x_3 \quad x_4]^T = [q_1 \quad q_2 \quad \dot{q}_1 \quad \dot{q}_2]^T$.

(10 marks)

4b) Using the feedback linearisation technique to design a controller for the two-link planar robot manipulator to ensure asymptotic stability.

(10 marks)

4c) Assign appropriate values to the static gain matrix (K) of your controller to track the reference signals

$$q_{1d}(t) = 0.5\sin(4t)$$
$$q_{2d}(t) = 0.5\cos(4t)$$

for joint positions. Using the system parameters $m_1 = 500gr,\ m_2 = 400gr, l_1 = 300mm, l_2 = 200mm,$ and $g = 9.81\ m/s^2$, illustrate the behaviour of the system for 10 seconds by Matlab.

i. Plot q_1, q_{1d} with respect to time on the same figure and comment on your observations

ii. Plot position and velocity errors with respect to time. Comment on what are the restrictions to increasing the static control gain in practical application.

(10 marks)

Reference

Baek, J. and Kwon, W., 2020. Practical adaptive sliding-mode control approach for precise tracking of robot manipulators. *Applied Sciences*, *10*(8), p.2909.