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Course outline

How does an NPTEL online course work? ()

MATLAB ()

Module 1: Introduction to MATLAB ()

Module-2: Building Your Code with MATLAB ()

Module 3: Errors and Approximations ()

Module 4: Linear

Week 4: Assignment

Assignment not submitted **Due date: 2023-02-22, 23:59 IST.**

Instruction to Students: In all problems, please report your answer accurate to four significant digits. If the solution is `0.0123456`, then please report either `0.01234` or `0.01235`.

Problem 1: Gauss Siedel

Consider the following set of linear equations:

$$\begin{aligned} -4x_1 + 2x_2 + x_3 - x_4 &= -11 \\ -x_1 + 5x_2 + 4x_3 + 2x_4 &= 4 \\ x_1 + 2x_2 + 5x_3 - 2x_4 &= 16 \\ 3x_1 - x_2 + 8x_4 &= 10 \end{aligned}$$

We wish to use Gauss-Siedel method for solving the above set of linear equations, starting with the initial guess, `x=[2; 0; 2; 3]`.

In each iteration, we compute all the four elements of the vector $\mathbf{x}$ (i.e., $x_1^{(i+1)}$ to $x_4^{(i+1)}$) using the procedure in the course videos.

Thereafter, please compute the error as well. The error after the n-th iteration is:
$\max \left|\mathbf{x}^{(n)} - \mathbf{x}^{(n-1)}\right|$,
i.e., the highest absolute difference between current and previous iteration values of vector `x`.

Please perform **four** and **twelve** iterations of the Gauss-Siedel method and answer the following questions.

1) Please report the value of `x` after **four iterations** of the Gauss Siedel method.

0.4 points

2) Please report the maximum error obtained **at the end of 4th iteration**. For example, if the solution in two iterations are stored as `x` and `xold`, then the error is given as: `err=max(abs(x-xold))`.

0.2 points

3) Please report the maximum error obtained **at the end of 12th iteration**. For example, if the solution in two iterations are stored as `x` and `xold`, then the error is given as: `err=max(abs(x-xold))`.

0.4 points

Problem-2: Gauss Elimination

For the system in Problem-1, perform Gauss Elimination to obtain the following matrix in the upper-triangular form

$$[A \mid b] = \begin{bmatrix} \hat{a}_{11} & \hat{a}_{12} & \hat{a}_{13} & \hat{a}_{14} & \hat{b}_1 \\ 0 & \hat{a}_{22} & \hat{a}_{23} & \hat{a}_{24} & \hat{b}_2 \\ 0 & 0 & \hat{a}_{33} & \hat{a}_{34} & \hat{b}_3 \\ 0 & 0 & 0 & \hat{a}_{44} & \hat{b}_4 \end{bmatrix}$$

Please answer the following questions:

4) Please complete all the steps of Gauss Elimination and report the **first row** of the matrix so obtained.

0.2 points

5) Please report the value of $\hat{a}_{22}$

0.2 points

6) Please report the value of $\hat{a}_{33}$

0.2 points

7) Please report the value of $\hat{a}_{34}$

0.2 points

Problem Solving Session ()

8) Please report the value of $\hat{a}_{44}$

0.2 points

Problem-3: LU-Decomposition

The linear equations in Problem 1 were written in the form *Ax=b*. Perform the LU Decomposition (using Gauss elimination procedure of the previous problem). The L matrix has the following form:

$$L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ \alpha_{21} & 1 & 0 & 0 \\ \alpha_{31} & \alpha_{32} & 1 & 0 \\ \alpha_{41} & \alpha_{42} & \alpha_{43} & 1 \end{bmatrix}$$

Please report the specific values of L matrix below.

9) Please report the value of α_{21}

0.2 points

10) Please report the value of α_{31}

0.2 points

11) Please report the value of α_{41}

0.1 points

12) Please report the value of α_{32}

0.1 points

13) Please report the value of α_{42}

0.1 points

14) Please report the value of α_{43}

0.1 points

15) Please solve the linear equation and report the value of x

0.2 points

Problem-4: Tri-Diagonal Matrix Algorithm

Please download and use the function myTDMA.m (https://drive.google.com/file/d/1TLFARSZr8yWsMQu6ZA5ndDdcsCfrSpXl/view?usp=sharing) for this problem.

We will formulate and solve a *transient* heat conduction problem in Week-9 of this course. Thereafter, in Week-10, we will learn to setup this problem. For now, let us say that the problem is already written as a tri-diagonal matrix. So, let's use TDMA method to solve the resulting linear equation. For the purpose of this assignment, you are expected to solve the following linear equation using TDMA:

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ \alpha & \beta & \alpha & 0 & 0 & 0 \\ 0 & \alpha & \beta & \alpha & 0 & 0 \\ 0 & 0 & \alpha & \beta & \alpha & 0 \\ 0 & 0 & 0 & \alpha & \beta & \alpha \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{bmatrix} = \begin{bmatrix} 120 \\ b \\ b \\ b \\ b \\ 25 \end{bmatrix}$$

where, $\alpha = 5, \beta = -10.5, b = -12.5$.

Our implementation of the tri-diagonal matrix algorithm to solve the above system of equations is provided in the uploaded file, `myTDMA.m` (https://drive.google.com/file/d/1TLFARSZr8yWsMQu6ZA5ndDdcsCfrSpXl/view?usp=sharing). Please use this to solve the above problem and report the results below.

16) Please report the value of vector x obtained from solving the TDMA.

1 point

You may submit any number of times before the due date. The final submission will be considered for grading.

Submit Answers