

ECE 480
Winter 2023
Prerequisite Assessment Test
“Write out and sign the Honor Pledge”

- (1) Find the result of the following complex expressions as magnitude and phase (degrees) as well as real and imaginary components:

$$A = 2 - j2, B = -2 + j2, C = 2 - j3, D = -5 - j4, E = 2 + j, F = j^j,$$

and

$$G = (D^2 E^2) F / A^3 B^2 C^4.$$

- (2) Consider a LTI system with the following transfer function

$$H_a(s) = N_a(s) / D_a(s),$$

where

$$D_a(s) = s^2 + (\Omega_0/Q)s + \Omega_0^2,$$
$$N_a(s) = k (s^2 - (\Omega_0/Q)s + \Omega_0^2),$$

and given the following information:

- The poles are given by: $1.256637 \times 10^5 \angle (\pm 95.73917^\circ)$.
 - The zeros are given by: $1.256637 \times 10^5 \angle (\pm 84.260829^\circ)$.
 - The DC magnitude response is equivalent to 12.0412 dB .
- (a) From the above information, find the following parameters: Q , ω_0 , and k .
- (b) Derive expressions for the magnitude and phase responses. Hence, plot it in order to determine the type of filter. What is it?
- (c) Plot the magnitude response in dB, and the phase response in degrees versus the frequency in Hz.
- (d) Find the frequencies at which the phase responses are:
- -47.2246 deg .

- -90 deg.

(e) Plot the group delay in msec.

(3) Find the first 6 terms of the Taylor's series expansion of the following functions:

(a) $y = \frac{1}{(1 + 0.5x^2)^{3/2}}$.

(b) $y = \cosh(x^{-1})$, $x \neq 0$. Hint: Write $y = \sum_{i=0}^{\infty} a_i x^{-i}$

(c) $y = 2.302585 \log_{10}(7.389056 - 3.694528x)$, where $|x| < 2$

(d) $y = a^x$, where $a = 20.0855$.

(4) Find the following:

(a) $\sum_{n=0}^{\infty} (-0.8)^{2n}$

(b) $\sum_{n=-4}^5 (0.85)^n$

(c) $\sum_{n=0}^6 0.5^{-n}$

(d) $\sum_{n=0}^8 n(n-4)$

(5) Expand the following signals in terms of their sinusoidal components. Hence, plot their line spectra and calculate their average power:

(a) $\sin(12\pi t + \frac{\pi}{3}) \cos(8\pi t)$

(b) $\cos^2(6\pi t + \frac{\pi}{3})$

(c) $\sin^5(12\pi t)$

(6) Solve the 2 simultaneous system of equations to obtain the real roots:

$$y - 50 \tan^{-1}(x/2) = 0$$

$$y - (x - 4)^2 = 0.$$

(7) Consider the following signal $x_a(t) = -5 + \sin^3(40\pi t) + 2 \sin(60\pi t - \pi) \sin(140\pi t)$

(a) Write $x_a(t)$ as the sum of 4 sinusoids, in addition to a dc component. Hence, plot it.

- (b) Plot the line spectrum of the signal $x_a(t)$.
- (c) Determine the average power of $x_a(t)$.
- (d) Consider the first order filter with transfer function given by

$$H_a(s) = \frac{s - \omega_0}{s + \omega_0},$$

where $\omega_0 = 200\pi$ rad./sec. Determine its magnitude and phase frequency responses. Hence, write the steady-state output response of the filter if the input signal is $x_a(t)$.

- (e) Determine the average power of the steady-state output.
- (f) Derive and plot the step-response of the above filter

Verify your answers using Matlab whenever possible.