Assignment 3

Hansen's model with two shifts

The economy is populated by a continuum of infinitely lived households who offer labor and consume c_t. Households work one of two work shifts or not at all. The shifts correspond to working only a straight time shift (h_1) or straight time plus overtime ($h_1 + h_2$). Total time endowment is normalized to 1. Let π_1 be the fraction of individuals that work only straight time and π_2 be the fraction that work straight time plus overtime. Household's preferences are given by

$$U(c_t, h_t) = \log c_t + \mu \log(1 - h_t),$$

where $h_t \in \{0, h_1, h_1 + h_2\}$ and μ determines the relative weight of leisure in the utility. The household's discount factor is β with $0 < \beta < 1$.

Output y_t is produced using the following production technology:

$$y_t = \gamma e^{z_t} k_t^{\alpha} n_t^{1-\alpha},$$

where k_t is beginning of period capital and n_t is total hours ($n_t = \pi_{1t} h_1 + \pi_{2t}(h_1 + h_2)$) . z_t is a random shock to productivity.

The resource constraint is

$$c_t + i_t \leq y_t.$$

The law of motion for the capital stock is

$$k_{t+1} = (1 - \delta)k_t + i_t,$$

where δ is the rate of capital depreciation and I_t is investment. Moreover the stochastic process follows

$$z_t = \rho z_{t-1} + \epsilon_t.$$

Social planner's problem

1. Write down the social planner's problem for this economy. (I recommend not trying to restate the problem in terms of total hours.)
2. Derive a set of first order conditions that characterize a solution to this problem.
3. Characterize the steady state for a nonstochastic version of this problem.
4. Linearize the Euler equation and two additional first order conditions of your choice.

Decentralized market solution

3. Decentralize this economy. State the maximization problem of the representative household and firm. (You do not have to derive first order conditions.)