

## 9 Recommender System

This assignment is dedicated to helping you understand SGD and recommender systems. You need to download the `ml-latest-small.zip` at <https://grouplens.org/datasets/movielens/>

### 9.1 Data Set

Note that the zip file contains side information (e.g. tag applications) that will not be used in the project: we consider only the ratings from the users. Therefore, the first step is to pre-process the data, and organize all the users' ratings as a matrix. Suppose there are  $n$  users and  $p$  movies. Then the size of the rating matrix  $M$  is  $n \times p$ . Let us denote the index set of observed entries by  $\Omega$ .

The second step is to divide  $\Omega$  into two sets  $\Omega_1$  and  $\Omega_2$ :  $\Omega_1$  for training and  $\Omega_2$  for testing. To this end, we randomly 90 percent of entries in  $\Omega$  to form  $\Omega_1$ , and  $\Omega_2$  consists of the remaining.

### 9.2 Learning

Then you will have to solve the following non-convex program to learn the prediction matrix:

$$\min_{U,V} F(U,V) := \frac{1}{2} \sum_{(i,j) \in \Omega_1} (M_{ij} - \mathbf{u}_i \mathbf{v}_j^\top)^2 + \frac{\lambda}{2} (\|U\|_F^2 + \|V\|_F^2) \quad (9.1)$$

where  $M_{ij}$  is the  $(i, j)$ th entry of  $M$ ,  $\mathbf{u}_i$  and  $\mathbf{v}_j$  are the  $i$ th and  $j$ th row of  $U$  and  $V$  respectively.

1. For a given index  $(i, j)$ , derive the stochastic gradient  $\frac{\partial F(U,V)}{\partial \mathbf{u}_i}$  and  $\frac{\partial F(U,V)}{\partial \mathbf{v}_j}$ .
2. Suppose  $\lambda = 1$ . Describe the update rule of SGD and implement it with Python. You can randomly initialize all  $\mathbf{u}_i$  and  $\mathbf{v}_j$ . Note that you need to carefully choose the learning rate.
3. Plot the objective value against the number of iterations, and summarize your findings.

### 9.3 Evaluation

After we terminate SGD, we will obtain the solution  $U, V$ . Our prediction matrix  $X$  is then given by  $X = UV^\top$ . We evaluate the performance of our prediction matrix  $X$  by root-mean-square error (RMSE):

$$\text{RMSE} := \sqrt{\frac{1}{|\Omega_2|} \sum_{(i,j) \in \Omega_2} (M_{ij} - X_{ij})^2}.$$

1. Record the RMSE for the choice  $\lambda = 1$ .
2. Now pick  $\lambda$  from  $\{10^{-6}, 10^{-3}, 0.1, 0.5, 2, 5, 10, 20, 50, 100, 500, 1000\}$ . For each value, learn and evaluate the your model. Plot RMSE against  $\lambda$  and summarize your findings.