**SET 7: From Newton to Einstein PHYS1001**
**Hand in: Week 10 (24 Nov 2021)**

## Question 1: Simple Harmonic Motion

**A**
Consider the differential equation:

$$\frac{d^2x}{dt^2} + 16x = 0$$

This equation models a mass-spring system on a horizontal frictionless surface. Here $x$ is the displacement of a mass $m$ of 0.25 kg from the equilibrium position.

a) Find the spring constant, $k$, and the angular frequency, $\omega$, of the oscillations of this mass-spring system. **{4}**

b) Write down the general solution $x$ of this differential equation defining all terms. **{2}**

c) Find the solution of this differential equation for the case in which the mass starts its motion with initial velocity equal to zero. **{2}**

**B**
a) A block on a frictionless table is connected, as shown in Figure 1a, to two springs having spring constants $k_1$ and $k_2 = 2k_1$.
- Write down the net force on the block in terms of $k_1$ , $k_2$, and the displacement $x$ of the block from equilibrium. **{3}**
- Hence find an expression for the block's oscillation frequency $f$ in terms of the frequency $f_1$ at which the block would oscillate if attached to spring 1 alone **{4}**.

b) A block on a frictionless table is connected, as shown in Figure 1b, to two springs having spring constants $k_1$ and $k_2 = 2k_1$.
- Write down the net force on the block in terms of $k_1$, $k_2$, and the displacement $x$ of the block from equilibrium **{6}.**
- Hence find an expression for the block's oscillation frequency $f$ in terms of the frequency $f_1$ at which the block would oscillate if attached to spring 1 alone **{4}**.

**Figure 1a**

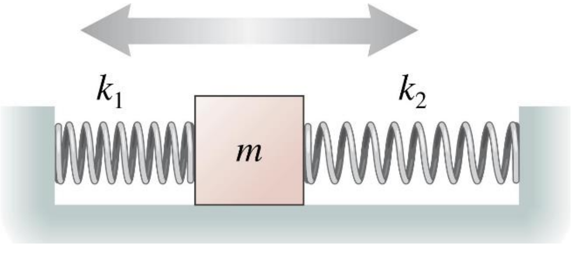
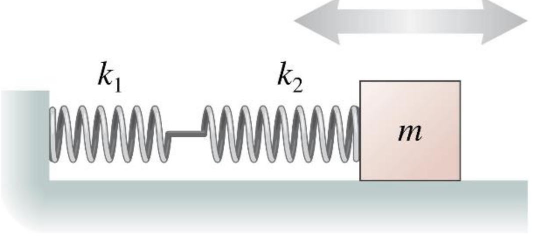



**Figure 1b**



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## Question 2: Oscillations

**A**

Consider the differential equation:

$$\frac{d^2x}{dt^2} + \frac{dx}{dt} + 25x = \frac{F}{m}$$

This equation models a mass-spring system with a driving force $F = F_0\cos(\omega t)$.

Here $x$ is the displacement of a mass $m$ of 0.25 kg from the equilibrium position, $\omega$ is the driving angular frequency and $F_0$ is equal to 1 N. By using complex notation, express the steady-state solution $x$ in complex form. Hence find an expression for the amplitude of the motion as a function of the drive angular frequency ω. {**10**}

**B**

Figure 2 shows a uniform rod of length $L$ = 1.0 m and mass $m$ = 5.0 kg pivoted at one end. The other end is attached to a horizontal spring with spring constant $k$ = 25 Nm$^{-1}$. The spring is neither stretched nor compressed when the rod is perfectly vertical. You can also assume that the force due to the spring is always horizontal.

a) Deduce an expression for the torque around the pivot point. Hence use Newton's second law to obtain the differential equation that describes the motion of the pendulum-spring system in terms of the angular displacement of the rod, $\theta$. **{8}**

b) Show that, in the small angle approximation the equation of motion is given by

$$\frac{d^2\theta}{dt^2} = -\left(\frac{3k}{m} + \frac{3g}{2L}\right)\theta$$

and hence calculate the rod's oscillation angular frequency $\omega$ and period T in the limit of small-angle displacements. **{7}**

**Figure 2**