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Learning from historical structures under compression for concrete 3D printing construction

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ABSTRACT

This paper reviews and extracts lessons from historic buildings, whose stability relies mainly on compression to resist gravity loads, that can inform the construction of affordable housing and shelters using 3D printed concrete without reinforcement and formwork. The first part consists of a literature survey of historic constructions with systems relying on compression considering four vectors of analysis: (1) form; (2) structural principle; (3) materials; and (4) construction process. The survey starts by identifying forms whose structural principle may be adequate for 3D printing of concrete applications. Then, historic structures displaying similar forms are analyzed in terms of structural behavior, the types of materials employed, and the construction process used to obtain foundations, walls, and roofs. A series of historically inspired shapes for printing is thus obtained from this survey. To address the printability of the structures identified after the survey, the second part of the paper provides a brief description of existing processes for construction scale printing considering material aspects, such as fresh state behavior, and printing system configurations. Addressing the fresh state properties of concrete is crucial as it determines whether the structure fails during printing. Finally, a set of strategies including potential toolpaths and intermediate states are defined to print the identified forms, considering issues concerning material requirements and printing process.

1. Introduction

According to current data [1], today's World population is around 7.8 billion people, with 56.2 % living in cities. By 2025, these figures are expected to reach 8.2 billion and 58.3 %, respectively. It is also estimated that 900 million live in informal settlements with inadequate basic infrastructure and degraded living conditions today and that this number will double by 2025. To solve the shortage of affordable housing, it is necessary to develop innovative construction techniques that can overcome current inefficiencies of the construction industry, while decreasing its ecological footprint. It is widely known that the construction industry has a great impact on the global economy. In fact, 13 % of world's GDP accounts for construction-related spending, which corresponds to $10 trillion. However, the sector's annual productivity in the last 20 years has only increased 1 %. The productivity gap is estimated to be $1.6 trillion a year [2]. In addition to lack of productivity, the building sector is one of the main contributors in energy consumption. Buildings consume around 40 % of the total primary energy use in the U.S. and E.U., which is partially explained by reliance on mechanical heating and air conditioning [3,4]. However, the energy consumption in buildings also includes embodied energy in addition to operational energy. For multistory concrete building frames in India, reinforced concrete accounts for nearly 90 % of embodied energy [5], which corresponds to the energy required to construct the building, including material extraction, manufacturing, and transportation [6]. Horizontal elements such as slabs are responsible for more than 60 % of the total embodied energy [7]. Another important aspect is the waste generated by the construction industry, which accounts for a third of the total waste in Europe [8,9].

The incorporation of additive manufacturing, or 3D printing, in the construction industry is a viable strategy for addressing the lack of productivity, as well as the energy and construction waste problems [4,10,11] and, therefore, for producing affordable housing. Despite its potential to change the way we build, challenges remain regarding the proper implementation for 3D printing using full-scale building materials. While the incorporation of reinforcement in 3D printing of concrete remains mostly in its experimental stage, the use of additive manufacturing to obtain compressive structures is already possible and should be considered a starting point to introduce this technology in the industry. Therefore, it is crucial to develop a framework to accomplish these

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first steps by identifying at first structural shapes that maximize compressive behavior, and then adaptable construction sequences from such examples.

Researchers can look to the past for inspiration on primarily compressive forms, as well as for construction sequences that can be adapted to robotic additive manufacturing. The historical constructions that relied on predominant compressive behavior can be divided into two categories from a material assembly standpoint: (i) based on the layering of discrete elements, such as mudbrick and stone masonry, in which the elements are connected by a binder that provides a source of adhesion; (ii) based on continuous deposition of fresh material, which takes advantage of fresh state cohesiveness of certain materials, such as reed. In both categories, formwork can be avoided. If such methods can be adapted for construction situations involving concrete and reduced formwork, there are many potential benefits.

The use of 3D printing technology in certain situations can reduce construction time, address a lack of specialized and technology-oriented workforce, reduce job accident rates, and promote a broader range of architectural shapes. Two possible applications of this technology are disaster relief situations, where construction time is crucial, and housing for low-income people. As there are significant opportunities to apply 3D printing for housing, this paper seeks general precedents based on forms and construction sequences that may be adaptable to the scale and context of housing.

While searching for precedents and possibilities, however, certain requirements for 3D printing must be considered. In the case of additive manufacturing of cementitious materials, the deposition occurs layer by layer, developing interlayer bonding. The bonding strength can be experimentally determined by means of the splitting tensile strength test. The bonding or interfacial strength in 3D printed concrete depends mainly on two qualities: moisture content on the surface and print-time interval. For moisture content, the surface of the layer should present enough workability to allow development of the bond between old and new layers. Lower moisture content is associated with lower inter-layer strength. The moisture content is affected by bleeding and evaporation rates. The pressure used in the extruder also plays an important role [10,12]. For print-time interval/Delay time, Lee et al. [13] observed that an interval of 15 min between printed layers led to a bonding strength greater than the tensile strength, while a gap of 30 min resulted in bond failure at the interface. This result is explained by the fact that larger delay times lead to a drop in adhesion, since workability is lost in the process. So, in additive manufacturing, if the delay time is controlled, the layers will not be independent as in the case of discrete elements of masonry. Layers are the result of a continuous printing process where they become adhered. Therefore, the thickness necessary for the geometries tends to be lower than in historical construction techniques. Despite these differences in behavior, the shared goal of efficient, primarily compressive shapes makes historical structures a valuable source of inspiration for the design of concrete 3D printed structures with prevalent compressive behavior. The first step in this regard is to analyze such historic structures and the employed techniques, which constitutes the main goal of this paper.

The paper is organized into four sections. Section 2 consists of a literature review of historical constructions with structural systems relying on compression behavior considering four vectors of analysis – form, structural principle, materials, and construction process used in foundations, walls, and roofs, with the goal of identifying appropriate forms for concrete 3D printing. In section 3, a review of existing concrete printing systems and material requirements, such as fresh state behavior, is presented, to address the printability of the forms identified in the survey. In section 4, a set of strategies including potential toolpaths and intermediate states to print the identified shapes are outlined, considering material and requirements and printing systems features. Section 5 summarizes the paper and presents the conclusions. Fig. 1 summarizes the main steps of the analysis followed in this paper.

2. Historic constructions

The history of architecture has many examples of building structures with prevalent compressive behavior that did not include reinforcement to complement the lack of tensile strength, which was the case in most early construction techniques, such as brick and stone masonry. The analysis of such examples can provide insight into the kind of structural forms to consider in the 3D printing of concrete buildings, as addressed in following.

2.1. Structural systems classification

Various structural classification systems can be found in the literature, including the one proposed by Engel [14], summarized in Table 1. Structures with primarily compressive behavior tend to rely on axial forces to transmit loads and, therefore, fall into two groups: active form systems and surface form systems.

Active form systems consist of flexible shapes that carry their own weight and cover the entire span [14]. Although they are not directly relevant to 3D printing, the inversion of tensile funicular structures

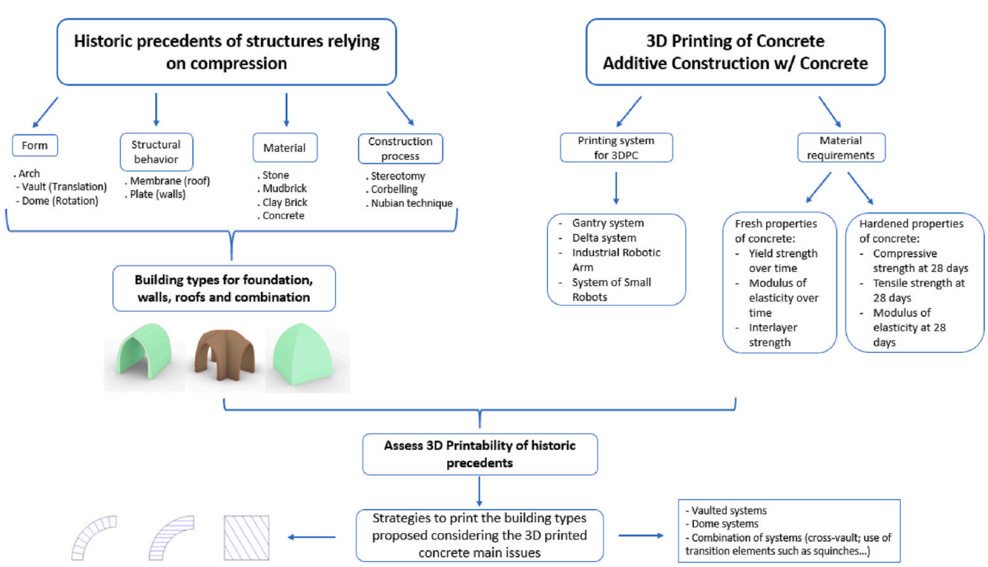
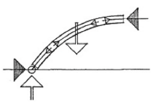
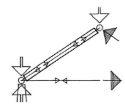
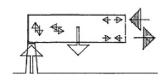
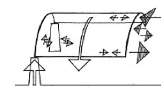
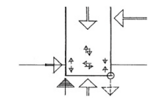


Fig. 1. Flow-chart of the problem statement and analysis.

Table 1
Structural systems classification, adapted from Ref. [14].

Structural groups	Active form	Vector form	Cross section form	Surface form	Height form
Type of forces	Compression/tension	Compression/tension	Bending forces	Membrane forces (in-plane)	Combined forces
Type of structures	Cable; Tent; Pneumatic; Arched	Trusses	Beams; Frames; Slabs	Plate; Folded plate; Shell	High-rise buildings
Load transfer					

leads to pure compression, such as in the catenary curve. Although some compressive contemporary structures are designed in this way, Adriaenssens et al. [15] mention that the catenary is not appropriate to use as the standard geometry for a barrel vault due to a rise in stress by a factor of 2.5 in the supports in comparison with a circular shape.

Surface form systems can be divided into plate or folded plate systems and thin-shell systems. In practice, plate systems correspond to bearing walls while thin shells correspond to the roof structure. Shell structures resist loads using a combination of bending and membrane action [16]. However, the type of structures pretended should minimize bending behavior and rely mostly on membrane action, thus sustaining loads by mostly axial and shear forces. Membrane action in a shell is then characterized by in-plane forces, or a plane stress state, and is described by the membrane theory of shells.

2.2. Historical forms typology

Vaults and domes are shells that can be obtained by translating or rotating arches, respectively. Vault variations include the barrel vault, which corresponds to the simple longitudinal translation of an arch. The blocks are assembled in the shape and size of the *voisseurs*, allowing the forces to be transferred around the arch (Fig. 2a). To restrain the arch from spreading, buttresses are required to absorb the thrust. In the case of masonry, the construction of a barrel vault with inclined courses avoids formwork, which saves the time and cost of assembling and deassembling a supporting structure. The structural behavior of the barrel vault is highly dependent on the ratio between the span and transverse chord width. A larger ratio leads to the cross-section working as a beam, in which case, the bending stiffness cannot be neglected. As such, the barrel vault needs to be studied as a shell, in which membrane and bending forces must be considered in the design of the structure.

The cross vault is obtained by intersecting two-barrel vaults in a right angle, which allows windows to be inserted in the side walls. The thrust forces that are absorbed by the groins are propagated to the four corners of the vault, as shown in Fig. 2b and c. The compressive forces P_{ϕ} distributed along the mid-sections equilibrate the thrust resultant from the shearing forces $P_{x\phi}$ acting on the side. The rib vault can be obtained by intersecting two pointed-barrel vaults, created by translating pointed or Gothic arches. The ribs serve to strengthen the groins, creating a supporting skeleton. In addition, since the arch is pointed, the impulse is lower than in a semi-circular arch.

Domes are formed from arches through rotation. According to Billington [17], in domes vertical gravity loads are carried by meridional arch-type forces, bending effects are neglectable along the meridians as long as the rings are uncracked, and it can be supported exclusively by vertical reactions if its base is stiff enough to absorb the horizontal thrust from the edge meridional force, for instance, by having a thicker ring at the base.

The hemispherical dome is formed by a 360° rotation of a semicircular arch (Fig. 3a). In this type of dome, hoop forces lead to compression at the top of the structure, preventing the dome from falling inward. Below approximately an angle of 52° from the vertical axis, the horizontal component of the resultant force will change direction, acting in tension (Fig. 3b and c). Therefore, the structure will tend to spread outside due to the deadload coming from the upper region [17,18].

The pointed dome is obtained in the same way as the hemispherical dome but using a Gothic arch instead. Since the thrust transfer is close

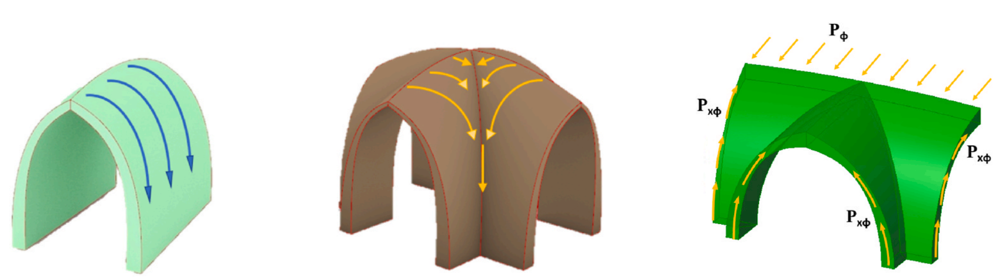


Fig. 2. Force transmission in vaults: (a) force transmission in a barrel vault; (b) force transmission in a cross-vault; and (c) membrane forces in cross-vault.

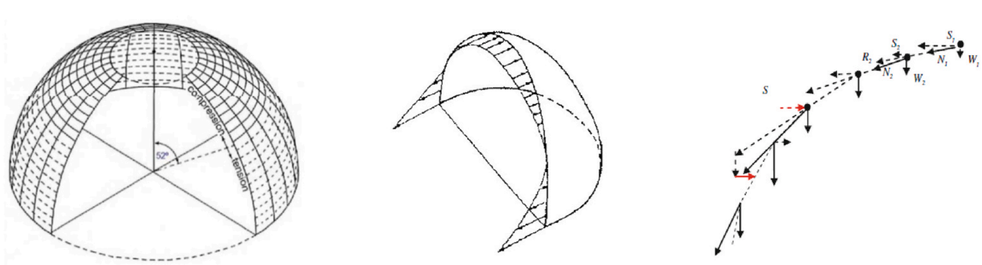


Fig. 3. Force transmission in domes: a) compression and tension rings in monolithic dome; (b) variation of stresses N_{ϕ} along the meridian section of hemispherical dome; and (c) membrane stresses [18].

to vertical, the impulse at the base is lower. However, the pointed region presents larger stresses.

2.3. Material considerations

Vernacular architecture evolves from the local availability of materials under the influence of cultural tradition. As such, it varies in accordance with the region, climate, and culture. Buildings sustained by the cultural values in a specific region reflect the identity of the associated community and it evolved over time by trial and error to meet people's needs [19,20]. In looking at vernacular architecture as a historic precedent, it is important to analyze the materials, structural forms and construction processes used in each culture [21–24], with different materials resulting in different structural and construction systems. In thin-shell structures with membrane action materials were usually assembled in layers or as discrete elements. The most popular materials employed in these geometries were adobe/mudbrick, clay/firebrick, and stone.

2.4. Historic precedents

After identifying shells with membrane action as the structural system with greatest potential for use in 3d printing, and the materials historically used for such structures, it is useful to focus on paradigmatic case studies. An important feature in all these cases is that form evolved to meet the need for maximizing compression, since the main materials employed – mud, clay bricks and stone – had low tensile strength.

2.4.1. Earthen construction

Earthen building systems can be found in a variety of locations, ranging for South America to Asia. The adoption of rammed earth and adobe is among the oldest vernacular systems [25]. Mudbricks result from mixing clay with straw and then dry them in the sun to harden the material. Beehive houses (Fig. 4a) are a paradigmatic example of dome construction in a mostly hot-dry climate with a broad diurnal and yearly temperature range [26]. The behavior of this structure consists of compressive hoop forces in the upper part, which prevents the bricks from falling inward, and tensile forces acting close to the bottom of the dome. Taq Kasra (Fig. 4b) is an example of a funicular structure that employs mudbrick as the main construction element. It is the largest brick vaulted structure in the world and is a landmark in Persian history. The funicular shape of the vault results in a compression-dominant behavior, which is adequate for the material employed.

2.4.2. Stone construction

Stone construction can be also found in a variety of locations and typologies. The most prevalent applications are stone masonry and stone carved constructions. In stone masonry, discrete stone elements can be assembled without binder (dry-stone) or using a binder/mortar. Two of the most popular applications of stone were in Ashlar masonry, where stones were cut in proper shape and joined using cement mortar, a process commonly used in the building of flat arches, and in Rubble masonry, where stones were cut in irregular sizes and shapes. A famous example of stone construction is shown in Fig. 4c.

2.4.3. Persian domes

Persian architecture is rich in material and engineering knowledge. From 8000 to 3000 BC, construction made a vast use of stone and mud brick to obtain bearing walls, foundations, and floors. Later, around 2000 BC, composite structures consisting of a combination of brick and mud brick marked the beginning of the use of arched roofs [27]. Besides hemispherical domes, Persian architecture also used bulb domes. Bulb domes present lower distributed stresses along the arch, comparatively to hemispherical domes, allowing for lower thicknesses. The top part of the bulb dome resembles a pointed dome, while the bottom part is like a hemispherical dome.

In addition to dome construction, corner construction was a striking feature in Persian architecture, which eventually led to two innovative dome solutions: the dome on squinches and the dome on pendentives. Both squinches and pendentives serve as support and transition elements between the square plan of the bearing walls system and the circular base of the dome. Squinches consist of filling in the angles of a wall base where a dome rests, thus spanning the corners of the planes to fit the dome. In fact, the first set transforms the square into an octagon, while the second set produces a hexadecagon (sixteen-sided polygon), which approximates the circle. According to Hejazi et al. [28], three main techniques are employed in Persian squinch construction (Figs. 5 and 6a), or *Sekunj*, namely: (i) *Filpush* – layers are created from the corners to bridge the angle between the rectangular sides of the base; (ii) *Patkin* – entails step-by-step projection of a wall [28]; and (iii) *Patkaneh* – consists of several rows of niches placed one over the other [28]. Pendentives consist of triangular segments of a sphere, whose purpose is to support the placement of the dome on the desired base, by creating a set

Fig. 4. Earthen and stone construction examples: (a) beehive domes, Harran, Turkey; (b) Taq Kasra, Old Persia, now Iraq; and (c) stone masonry wall, Macchu Pichu, Peru.

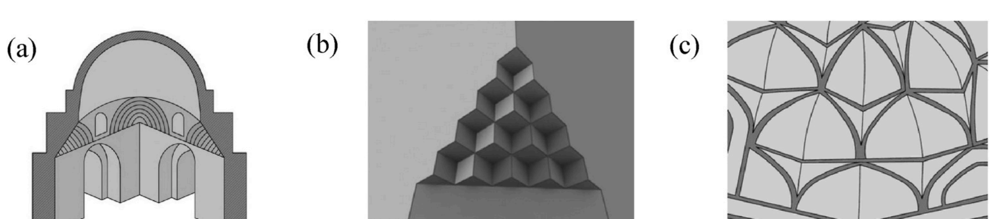


Fig. 5. Types of Persian squinch (or Sekunj) techniques: (a) Filpush; (b) Patkin; (c) Patkaneh [28].

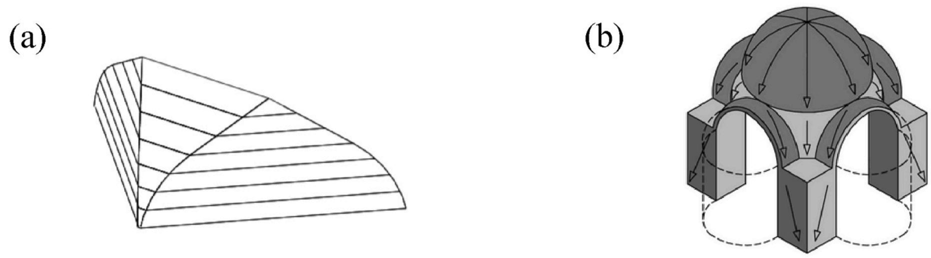


Fig. 6. Transition between different curved and planar surfaces in dome construction (a) squinch in Persian domes; and (b) pendentives in Western domes [28].

of curves that arch inwards to encounter the dome's base, as shown in Fig. 6b.

2.4.4. *Round and Gothic vaulting*

Concrete vaulted constructions from ancient Rome displayed large spans and a wide variety of solutions, ranging from domes to cross-vaults, where the round arc was the prominent element. Concrete vaulting was accomplished by adopting a wooden centering process, where truss elements were widely used [29]. The Colosseum and the Pantheon of Rome are among the most inspirational structures from ancient Rome (Fig. 7a and c), where vaulting ribs were employed to propagate loads between arched elements. Unlike Gothic ribs that were visible after construction, ribs built into Roman concrete vaults, consisting of stone or brick arches, ended up hidden after decoration. Travertine ribs were also adopted to increase the stiffness of barrel vaults and receive the loads from the brick relieving arches of the walls (Fig. 7b). This system assured an efficient load propagation to the foundation.

Roman architecture was precursor to the Romanesque, which was the first artistic style to be widespread in the West [30,31]. Round arches made with stone or clay brick, culminating in barrel and groin vaults were used to support the roof, with thick walls and piers to absorb the outward thrust of the vaults. The Gothic style came after the Romanesque architecture and was characterized by the pointed arch, which led to the ribbed vault. Canterbury Cathedral is an example (Fig. 8a) of the Gothic style [32]. The fan vault was a type of rib vault used in England, in which the ribs are equidistant and display the same curve (Fig. 8b).

The introduction of the pointed arch led to a great advance in vault design. This geometry reduced the distributed stresses along the arch, resulting in larger interior spaces and taller construction with thinner walls [33,34]). In addition, since the transferred thrust is close to vertical, lower impulses are generated at the base of the structure. A famous example is the nave of the Notre-Dame's cathedral in Paris, where rib vaults and flying buttresses were widely employed (Fig. 9a). Flying buttresses provide stability for lateral loads from pressure of the vaults and the wind [34]. Another remarkable example of late Gothic architecture is the Church of Santa Maria de Bélem in the Jerónimos monastery in Lisbon (Fig. 9b), where ribbed vaults permitted the construction of an almost flat ceiling. The rib's arrangement (Fig. 9c) translates into a compression-only network of forces, which intrigues the research community until today [15].

2.5. *Construction elements*

Following a discussion of relevant forms, this section reviews the processes used for building the three main construction elements – foundations, walls, and roofing system – in structures with prevalent compressive behavior, to extract relevant lessons for concrete 3d printing applications.

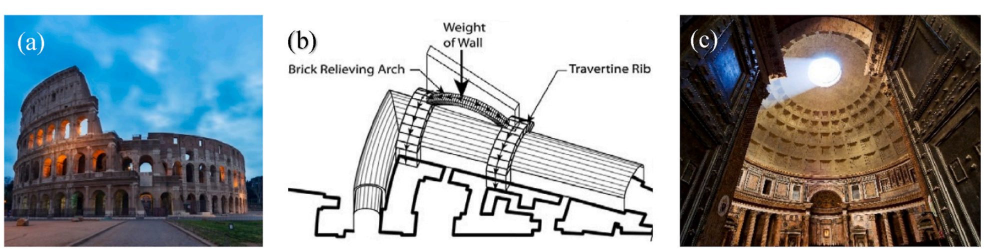


Fig. 7. Round vaulting: (a) Colosseum; (b) segment of the Colosseum [29]; and (c) Pantheon, Rome, Italy.

Fig. 8. Gothic vaulting: (a) main nave and (b) fan vault at the crossing, Canterbury cathedral, England.

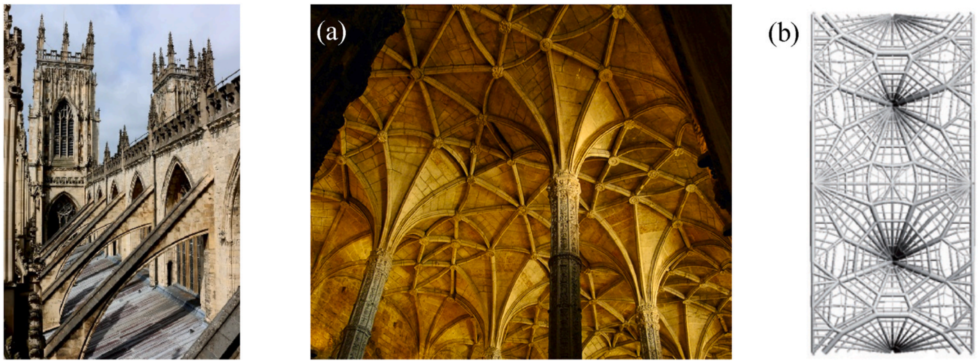


Fig. 9. Gothic vaulting: (a) flying buttresses, Notre-Dame, Paris, France; and (b) interior and (c) plan view of ribbed vaults, Jerónimos Monastery, Lisbon, Portugal [15].

2.5.1. Foundations

Foundations can be divided into shallow foundations and deep foundations. Shallow foundations are adopted when the soil bearing capacity close to the surface is large enough to resist the structure's loads. On the other hand, when the soil bearing capacity is not sufficient, deep foundations may provide friction resistance and/or bearing on stiffer soils or bedrock.

2.5.1.1. Shallow foundations. Shallow foundations found in historical constructions can be divided into three types: (i) footings – isolated foundations that concentrate loads from above; (ii) strip foundations – linear foundations created by digging trenches under walls; and (iii) raft or mat foundations – continuous slabs built on the soil. The selection of the foundation type depends on two main variables: (i) type of structural element and (ii) loading. The foundation for an isolated member, such as a column, requires an isolated footing, while a wall requires a strip foundation. However, if the magnitude of the loads is larger than the soil bearing capacity, larger foundations will be required to reduce the pressure on the soil by distributing the load over a larger area.

Isolated footings are found in several vernacular architecture cases, such as in Ghana's indigenous architecture, where stone was a vital component in footing construction. In humid climates, a strong footing that does not dissolve under water pressure is necessary [35]. In English vernacular architecture, all mud buildings require footings made of stone or brick with an impermeable layer to prevent water penetration or wicking that can transfer dampness to the structure. Individual footings consisting of stone elements with lime mortar [32]. In addition to stone, brick was also a popular solution for foundations that was initially made with sun-dried mud bricks and later with fired bricks [36].

In contrast, Strip Foundations were largely implemented across several other cultures. In Persian adobe construction, field stone was used to fill the excavated volume in combination with grout made of lime concrete. In addition, bamboo cane was used to connect adobe masonry walls to the foundation, providing ductility to the system [28]. Similar solutions can be found in Ancient Greek Architecture, where builders accounted for seismic movements, by incorporating logs and metallic cramps to connect the walls to foundation [37].

Raft or map foundations were popular shallow foundations in Ancient Egypt in the form of sand-box foundations, where the stone masonry platforms of the structure would rest on a thick-layer of sand. This solution allowed a stable foundation in case of flooding of the Nile River, and a large area to distribute loads, even in the absence of bedrock. A similar solution was implemented in Ancient Greek temples, where a large stone mat serving as foundation was placed on a sand bed [38]. Besides stone and brick masonry, concrete was a valuable foundation material, especially in Ancient Rome. Roman builders built concrete foundations using timber formwork such as wooden boards and posts. Solutions ranged from isolated footing for piers to mat foundations for temples [37,39].

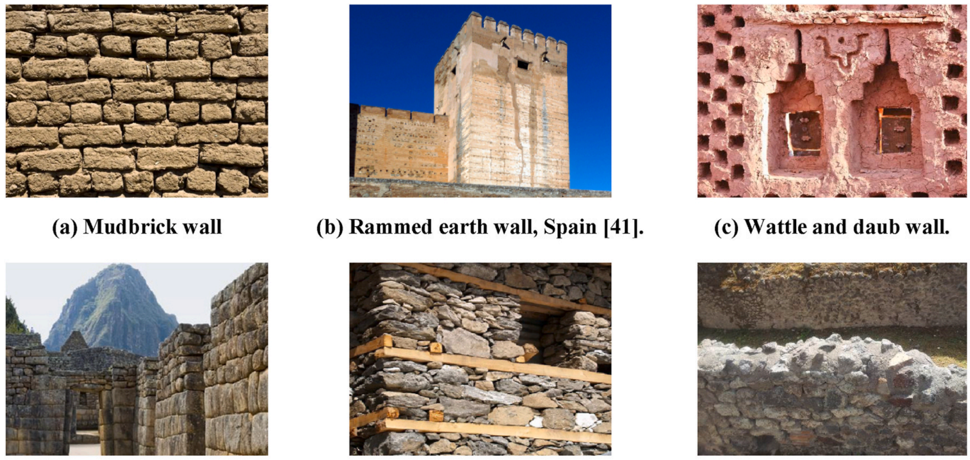
2.5.1.2. Deep foundations. When the soil is very weak and does not present enough bearing capacity, a deep foundation is required to transmit loads to a stiffer layer. Apart from some Roman construction examples, where a variation of a pile solution was used, this type of foundation was rarely used until the 18th century, when a solution combining piles and timber grillage was employed, for bridge pier application, which spread across several areas of Europe. Venetian and Amsterdam buildings also adopted piles due to the presence of very soft soils and water [37].

2.5.2. Walls

The materials and techniques adopted for wall construction differ in different regions of the world [25]. According to Zhai & Previtari [19], specific climate zones showcase a broad range of materials used in walls, presenting a diversity of materials that include bamboo, packed earth, wood and stone. Starting with earth construction, it can be divided into mudbrick (or adobe) and rammed earth. Mudbricks are prepared with clay and straw and then sun-dried (Fig. 10a). It can be obtained from a large range of soils and does not require expensive equipment and highly qualified labor, which drastically reduces operational costs. In addition, this solution provides good fire resistance and sound isolation. Another earthen construction technique is rammed earth, in which walls are built by ramming soil between wooden forms, which are moved upwards to build another section successively (Fig. 10b) [40,41]. An less durable alternative to rammed earth is a wattle and daub technique [23], which consists of building walls using vertical wooden stakes and wattles and filling the space between wattles with mud (Fig. 10c).

Stone masonry in walls is predominant in several regions across the globe due to large availability, strength, and weather resistance. When the wall consists of stones that were cut in regular dimensions and joined with mortar, an Ashlar stone wall is obtained (Fig. 10d). Another way to build with stone masonry is by using irregular shaped stones, or rubble stones. The vernacular construction technique "Bakhar" that is employed in the Himalayan regions of India and Pakistan is an example of combining the practicability of dry-stacked rubble stones with timber bands to provide tensile strength and confinement [41]. Fig. 10e shows a wall built with the "Bakhar" technique.

Another option largely employed in Roman construction was concrete, which at that time used pozzolana or gypsum as binders along with sand and brick rubble or rock as aggregates (Fig. 10f). Since Roman concrete was normally faced with brick or stone, several techniques were employed, from *Opus incertum*, which consisted of irregularly shaped uncut stones inserted in a concrete core, to *Opus reticulatum*, where diamond shaped bricks were used instead of irregular ones. The Romans also used formwork in concrete wall construction [39].



(a) Mudbrick wall
(b) Rammed earth wall, Spain [41].
(c) Wattle and daub wall.
(d) Dry-Ashlar stone wall, Macchu Pichu, Peru.
(e) Rubble stone masonry wall with wooden bands.
(f) Pozzolan-lime-rubble concrete wall from Pompeii.

Fig. 10. Types of wall construction observed in vernacular architecture.

2.5.3. Roofing system

In vernacular dwellings, the materials used in roof construction were mostly turf, wood, stone, and adobe brick or fire clay bricks. The former two materials are not adequate references for 3D printing, but the latter two are. Two types of roofing systems were usually used with these materials: flat and vaulted. Flat ceilings were mostly used in multi-story dwellings, whilst vault ceilings were implemented in single story dwellings, especially in hot and humid climates, where air stratification through buoyancy was desirable [19]. There are five main vaulting techniques: (i) Centering, (ii) Corbelling, (iii) Nubian vaulting, (iv) Gothic vaulting, and (v) Catalan vaulting. These techniques are described below. Table 2 describes the construction process and materials employed in each roofing system.

2.5.3.1. Roman vaulting. Roman vaulting employs the concept of centering, which is a type of formwork used to create arches, and it is one of the oldest processes used to create vaulted structures. These are built by placing wedged discrete elements in a radial orientation and until the keystone is inserted the arch does not have strength to support its self-weight, requiring formwork to advance construction. Centering can be used to build both pointed and barrel vaults and was extensively used in Ancient Rome to build vaulted roofs covering large spans, since it can provide a solution to construction with continuous deposition of fresh material (Fig. 11) [29].

Table 2
Simplified classification of roofing systems in historic constructions.

Roofing system	Roman	Corbeling	Nubian	Gothic	Catalan
Process	Elements placed in a radial orientation. Use of centering	Elements deposited on top of each other in a circular path.	Inclined courses of masonry against a previous course	Masonry elements fill spaces between wooden ribs.	Successive overlapping of thin masonry elements. Starts at corners, joining at the center.
Formwork	Yes	Yes/No	No	Yes	No (Yes for more complex shapes)
Materials	Clay bricks Stone blocks Roman Concrete	Adobe bricks Stone bricks	Adobe bricks Stone bricks	Adobe bricks Stone bricks	Tile (Stone; Clay brick)
Structural principle	Roof: Membrane action (Compression/Tension + Shear) Wall: Plate (Combined Bending and Axial)				

2.5.3.2. Corbelling. Another widely spread method to enclose buildings is Corbelling. In this technique, masonry elements are successively placed in layers on the top of each other, with cantilevered regions to form an arch shape (Fig. 12a). An empirical way to obtain stable layers is by adopting corbelled angles larger than 65° [42], since this minimizes cantilevered regions and promotes the projection of the center of mass to be over the base, which represent stability for deadload. An example of this type of solution is the building called *cardenha* found in Vale de Poldros, Portugal, whose walls and roof were built using dry-stone masonry [43,44] (Fig. 12b to d). The structural integrity in dry-stone applications comes purely from compressive forces and the interlocking of the stones, since no mortar is used [45]. To avoid collapse during construction, a second row of stone elements would be placed as a counterweight solution to maintain the projection of center of mass on the base of the system. Another case where dry-stone corbeling was employed is the Italian *trullo* houses in Apulia, whose conical roofs were constructed in two layers. A first layer of limestone pieces was laid, followed by an outer layer of limestone tiles, that were tilted outwards to drain the rainwater. Two main housing typologies were adopted: (i) circular walls leading to a singular conical roof, and (ii) square walls. With square walls, the process to enclose the structure starts with the construction of the exterior and interior walls, including windows and doors built using lintels or semicircular arches. Then, by adopting squinches, the square rooms become circular, and the domes could be constructed.

2.5.3.3. Nubian vaulting. The extensive use of formwork in centering solutions led to the search for alternative ways for enclosing roofs. A technique developed in Ancient Nubia and later rediscovered by Egyptian architect Hassan Fathy, who named it Nubian vault, permits the construction of vaults without the use of formwork. By adopting a vertical surface as a backrest to inclined courses of brick, the forces from the temporary structure would be transferred with minimal tension and bending. This technique is based on two main features: (i) inclination of the brick path against a backrest wall, and (ii) optimization of the vault's cross-section to maximize compression, which is possible by adopting a catenary shape [48,49] (Fig. 13).

2.5.3.4. Gothic vaulting. The use of formwork permits more complex and creative designs. In gothic vaults, ribs were used as main structural elements (Fig. 14a and b). The masonry ribs were built over wooden frames, and stone or brick masonry filled the gaps in between ribs with different layering patterns (Fig. 14c). Then, when the mortar dried, the framework was removed. Rib vaults can be divided into three main

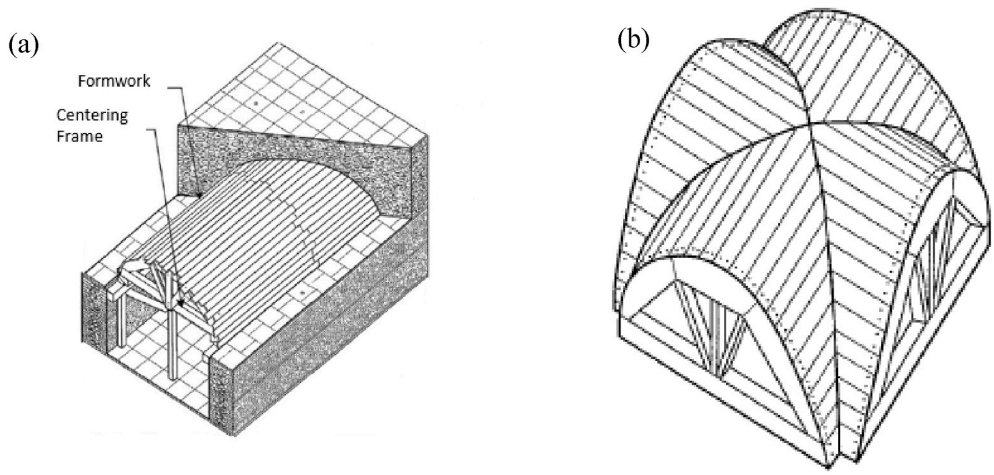


Fig. 11. Vault construction with centering: (a) concrete barrel vault and (b) concrete cross-vault [29]. The image illustrates how traditional, non-printed arched structures are made using wooden formwork made by joining thin boards, which may resemble layers of a 3D printed structure. The formwork supports the actual structure, which can be made by layering bricks or stones, or by casting concrete.

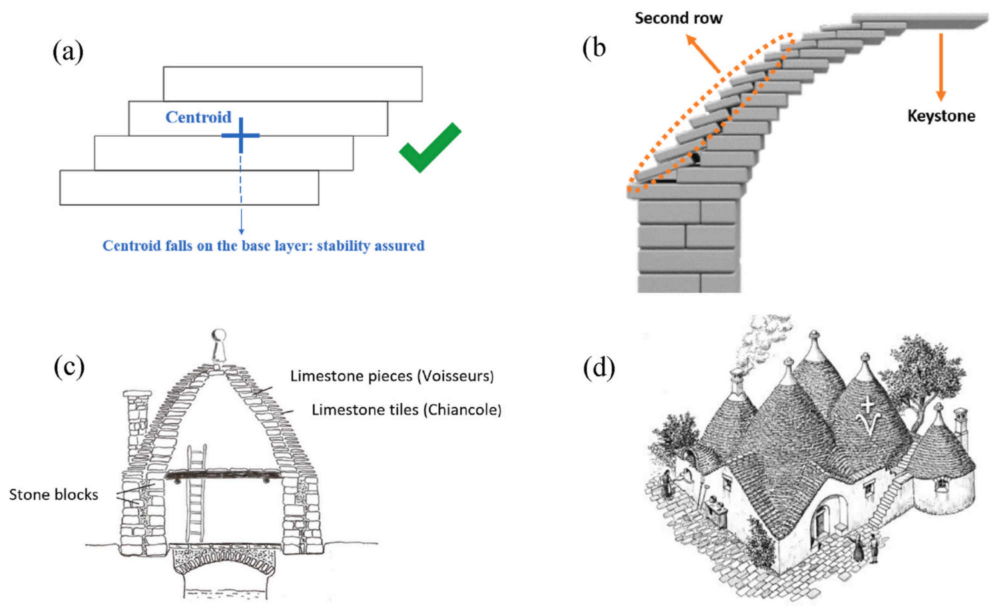


Fig. 12. Vault construction with corbeling: (a) diagram showing how brick stability of guaranteed, (b) section of a *cardenha* [44], (c) section of a *trullo* [46], and (d) complex *trullo* [47].

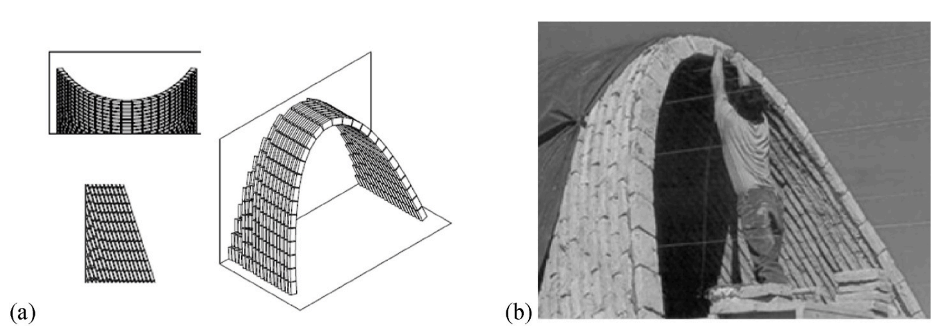


Fig. 13. Nubian vaulting: (a) construction diagram [50] and (b) craftsman building a vault [28].

groups: (i) sexpartite vaults; (ii) quadripartite vaults; and (iii) fan vaults. In a sexpartite vault, each bay is divided by two diagonal ribs and three transverse ribs, forming six parts. The ribs work as the skeleton of the ceiling structure, carrying the weight of the infill composed of small elements of stone. A quadripartite vault is divided into four parts, allowing a more even distribution of the loads to the supporting piers, thus allowing thinner walls. The quadripartite vault has single curvature [51,52]. Finally, the gothic influence in England was marked by the introduction of the fan vault, which consists of ribs with the

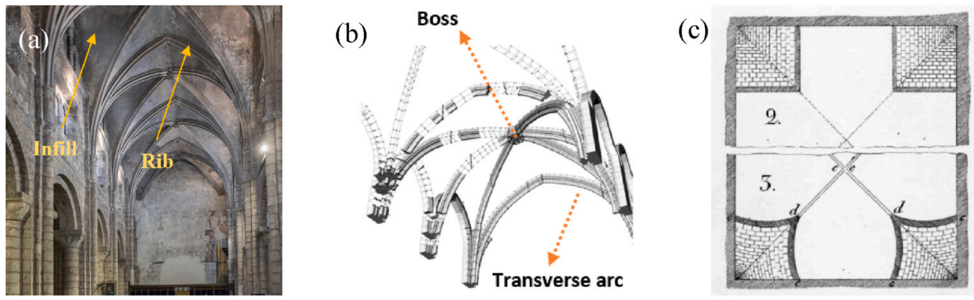


Fig. 14. Gothic vaulting: (a) main construction elements, (b) rib network configuration [55], and (c) different infill patterns (Lassualx, 1829).

same curvature, equidistant to the vertical axis of the pier, presenting a conoid geometry and resembling a fan.

Fan vault construction is divided into several steps. First, the walls and roof are built with formwork. Then, the arches that divide the roof into compartments are erected, and the conoids are assembled one horizontal level at a time. Finally, the boss that works as the keystone of the vertical conoid arches, is placed at the top, with the successive removal of formwork (Fig. 15). Rubble fill is usually placed inside the volume at the base of the conoid to reduce distributed loads caused by the thrust from shell to walls [53,54]. The transverse arches transmit most of the structural load at the pendant, carrying the thrust to the walls and buttresses through the conoids.

2.5.3.5. Catalan vaulting. One of the most widely spread methods employed to build structures without support elements is the Catalan vault. The Catalan vault is a masonry technique included in *tabicada* construction, which presents three main features: use of bricks, gypsum mortar, and no formwork. This building technique has its origins in Roman and Arab-Islamic cultures (as an adaptation of the Nubian vaulting) which was extended to other regions of the world, but perfected in Catalonia, Spain, and brought to the United States by the Catalan architect and builder Rafael Guastavino [20,56]. The construction process of the Catalan vault consists of successive overlapping of thin bricks using a mortar (cement, lime, gypsum). The brick laying starts at all corners and joins at the center of the ceiling, as shown in Fig. 16.

This vaulting technique presents similarities with the squinch transition techniques from Ancient Persian architecture, showing potential to be used in building pointed domes from squared bearing wall systems.

The historic examples above had an impact on Modern and Contemporary structures, which also constitute important precedents for concrete 3D printing applications. The technology of reinforced, pre-, and post-stressed concrete allowed for more complex shapes than before. One of the most famous concrete shell designers was Heinz Isler, whose work employed physical pre-modeling techniques to build these structures. One way to obtain valid shapes without excessive calculations was by mimicking natural structures [57,58]. Other remarkable shell structures include L'Oceanogràfic of Valencia (Spain) from Félix Candela, which showcases the great flexibility provided by concrete. The groin vault used in this building works predominantly under compression and it was built with minimum reinforcement, solely to control temperature and creep effects. The form finding algorithms and optimization methods employed in contemporary and modern thin-shell structures showcase the potential for unreinforced solutions, serving as an asset to the design of 3D printed concrete structures, since both material constraints and phased construction stand as the most determining factors in the design stage, and addressed further below.

2.6. Main lessons from precedents

The analysis of historic precedents was performed to extract lessons regarding forms and construction processes to obtain compression-dominant structures with 3d printed concrete. From the analysis above,

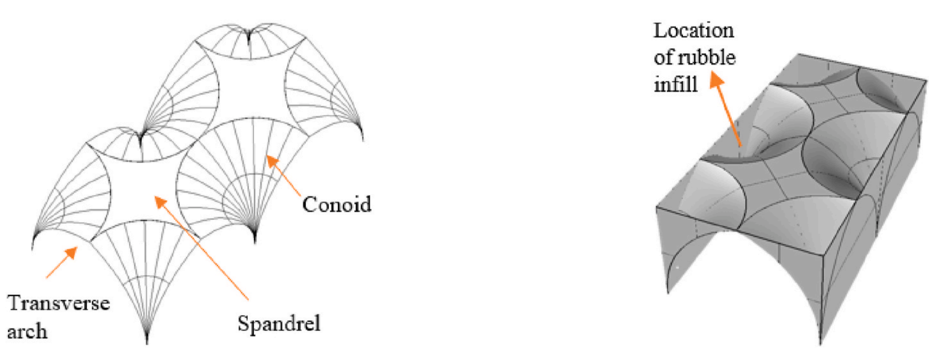


Fig. 15. Fan vault: (a) main construction elements, (b) Perspective view.

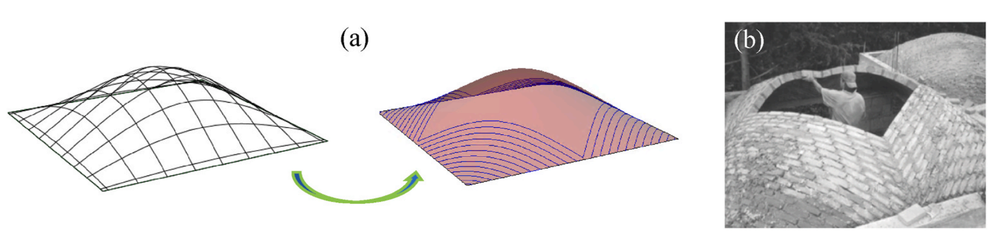


Fig. 16. Catalan vaulting: (a) construction diagram and (b) craftsman building a vault.

structures built with materials such as stone or adobe, that inherently do not have tensile strength, present systems consisting of load-bearing walls and shell structures such as vaults and domes. A summary of the lessons learnt from historic precedents is presented in Table 3, for each of the main construction elements.

From a 3d printing standpoint, where the material is deposited in continuous layers, the construction strategies presented for the arched roofing system of historic precedents can be summarized into: (i) Roman/Radial; (ii) Corbeling/Horizontal; (iii) Nubian/Inclined. Roman printing would require the nozzle to be tangent to the curve of the vault, in a radial orientation, leading to layers with wedged sections (Fig. 17a), in contrast with Corbeling and Nubian printing that would require the nozzle to be perpendicular to the layers, which are respectively horizontal and diagonal, resulting in layers with cross-sections of constant height (Fig. 17b and c). How these printing strategies can be applied to obtain forms with concrete printing technology will be further discussed in Section 4.

3. Concrete 3D printing developments, systems, and materials

Additive manufacturing, or 3D printing, is the process of extruding layers of material to form a three-dimensional object through a nozzle following a toolpath generated after a 3D model of the shape to print. After reviewing historical precedents and identifying appropriate forms for 3D printing of concrete, it is necessary to understand how these can be printed. There are two aspects that need to be considered. The first aspect is the type of printing system, which determines the printing envelope and constrains the size and geometry of the printed forms, and the second aspect is the printing material, whose rheological and mechanical properties in fresh and hardened states restrict both toolpath design and the printing process. Before these considerations, a discus-

Table 3
Summary of the analysis of historical constructions with prevalent compressive behaviour.

	Form	Structure	Material	Structural Principle	Construction process
Foundation	Shallow/Square	Footing	Stone; Brick; Concrete; Timber (Formwork)	Thick Plate (Axial; Shear; Bending)	Excavation, followed by formwork to group stone, clay brick or concrete blocks in the desirable form.
		Strip Foundation			
		Mat Foundation			
	Circular	Pile (Deep) Foundation	Timber	Beam on elastic foundation (Axial; Shear; Bending)	Hole made by manual drop hammers and placement of wooden pile.
Walls	Rectangular	Planar wall (w/w/o openings)	Stone; Concrete; Clay Brick; Adobe	Plate (Axial; Bending)	Corbeling
	Curved	Curved wall .(w/w/o openings)			
Roofing system	Planar (Flat)	Slab	Timber; Mix	Plate (Bending; Shear)	Centering
	Single curvature surface	Barrel vault (semi-circular and funicular)	Stone Adobe Concrete	Shell (Membrane + Plate)	Nubian
		Hemispherical Dome			Corbeling/Centering
	Combination of single curvature surfaces	Pointed barrel vault	Stone Clay brick		Nubian
		Cross vault			Gothic vaulting
		Pointed Dome	Clay brick Adobe		Gothic vaulting
			Dry-stone	Compression Interlocking	Corbeling
	Conoid	Fan vault	Stone Clay brick	Shell	Gothic vaulting
	Warped/Double curvature/Free-form surface	Catalan vault	Tile (Stone; Clay brick)	Axial (Compression)	Catalan vaulting

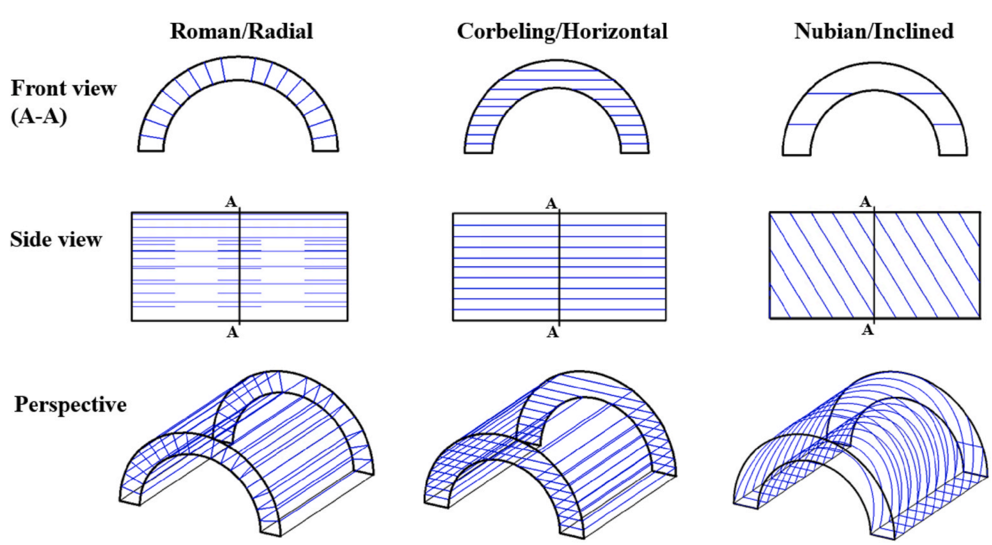


Fig. 17. Main layering strategies: (a) Roman/radial, (b) Corbeling/horizontal, and (c) Nubian/inclined.

sion on the main developments in concrete 3D printing is provided below.

3.1. Concrete 3D printing developments

3.1.1. Wall systems

Concrete 3D printing goes back to 2004, when Khoshnevis published a comprehensive paper on Contour Crafting (CC) [59], describing the technology as an extrusion-based gantry system that incorporates two trowels that assure smooth and accurate surfaces. Contour Crafting makes vertical walls by printing first the boundary walls and then infill patterns to strengthen the wall system. The printed walls serve as integrated formwork for cast concrete that is poured later. This system inspired other systems based on concrete extrusion, which accounts for most of the research in concrete 3D printing.

Research on concrete printed walls focuses mainly on structural performance, buildability, and failure modes during printing, which stems from elastic buckling and plastic collapse. To address elastic buckling, researchers have developed various methods to predict critical buckling length, from parametric mechanistic models [60], to finite element modeling [61], and other numerical procedures such as matrix methods [62], which have been confirmed by printing experiments. For infill patterns, truss pattern has been documented as preferable to diamond pattern because of lower strain and deflection for compressive loads, which is explained by the bracing effect at the intersection regions of the truss with the wall, thus reducing buckling length [63]. Plastic collapse of walls has been predicted by time dependent models based on Mohr-Coulomb failure criteria and subsequent validation by printing experiments [64,65]. In addition, Furet et al. [66] introduced thermal performance in the design of walls by developing a process that prints two polyurethane foam walls, that encase a third wall made of self-compacted concrete. This method was validated with the construction of a residential house in Nantes, France.

3.1.2. Enclosures: vaults and domes

Traditional formwork methods comprise wooden (solid) formwork as temporary support for flat roofs, pneumatic formwork for curved roofs [67], and backfilling with sand such as in construction with slab on grade. The viscous behavior of printed concrete challenges the fabrication of cantilever elements, which motivated innovative methods such as flexible molds for 3D printed shells [68], which act as formwork, and 3D sand-printed floors with varied rib geometries from compression-only form finding algorithms [69], which serve as integrated formwork. These solutions can be implemented in the context of concrete 3D printing but would hinder a full automated process that is envisioned in this paper.

When formwork is not desirable, overall shapes should either be redesigned to consider constraints such as maximum overhangs, or as suggested by Carneau et al. [70], obtained by modifying printing settings such as bead height or inter-layer time for every new layer. In fact, the authors fabricated a dome structure with inclinations exceeding the maximum overhang by optimizing printing settings. In addition, a concrete vault without formwork was fabricated as a proof of concept by printing a Nubian vault where the main part was obtained with a 40° angle, and the transition region with an inclination ranging from 0 to 40°. In Nubian vaulting, extrusion is perpendicular to plane of the layers, thereby maximizing the contact surface between layers and interlayer bond.

3D printing in construction extends beyond concrete, as the use of other materials such as clay and minerals is also under research. Clay printing has received attention for the potential in building tiny houses, such as the dome-shape *Tecla* houses made with local materials [71]. Another example is provided by the company Desamanera that uses minerals and natural binders for large scale 3D printing, allowing customizable finishes [72]. These initiatives push researchers to keep developing parametric algorithms for toolpath generation and studying manufacturability settings [73–75].

3.2. Concrete 3D printing systems

The past ten years saw technological advancements that were translated into innovative printing systems. For extrusion-based 3D concrete printing, the following systems have been adopted in both research and industrial applications: (i) gantry system; (ii) delta system; (iii) stationary industrial robotic arm system; (iv) cooperative small mobile robots; (v) mesh mold system. The gantry system (Fig. 18a) consists of a large frame structure that mimics smaller printing systems. It has been used to print contour wall forms layer by layer, which are then filled with regular concrete [59]. The main disadvantages of a gantry system are the following: (i) reduced flexibility, as it only allows three main movements, namely translation in x, y and z directions, and even tough additional features such as rotating nozzle have been added, it is still not as flexible as other systems; (ii) large size, as the gantry needs to be larger than the structure, which becomes specially difficult in multi-story building construction; (iii) high installation costs, as it needs to be assembled and calibrated for each use [76]. The delta system is an alternative to the gantry system, where the nozzle hangs from cables that can be raised and lowered to guarantee movement of the nozzle along the x, y and z directions, although in a non-linear fashion. It has the benefit of providing increased printing speed by using a lighter printing head, and a more efficient use of the printing space, due to allowing circular paths. In addition, since motion is non-linear, a larger printing area can be reached. A disadvantage of this system is alignment failures during printing that result from the smaller inertia of the printing heads. The printing envelope for each system is highly dependent on the technological advancements. The Gantry system from the company *COBOD* has a printing envelope of 6,8 × 7,7 × 5,8 (width x diameter x Height) meters [77], while the largest delta system from the company *Wasp* presents a height of 4.05 m, and an arm length of 3.30 m (Fig. 18b).

The previous printing systems present the common problem of scalability, as they are required to be larger than the structure to print. Alternatives include arm-based systems, such as stationary industrial robotic arms (Fig. 18c) and cooperative small mobile robots (Fig. 18d). The 6-axis industrial robotic arm system permits three translation movements and three rotations, which considerably increases printing flexibility. A disadvantage is the limited reach of the robotic arm, which might require the robot to be moved after printing structures of a certain size, adding complexity to the logistics and toolpath design. A system of small cooperative mobile robots avoids the scalability issue, since each robot is able to move on a defined region in a shared environment with other robots and contribute as a whole to complete a larger task [78]. However, since terrain conditions on construction sites tend to be irregular, the mobility of the robots can be compromised. Another system that can be used in additive manufacturing of concrete is mesh mold (Fig. 18e). This system involves spraying concrete over a printed mesh. One of the main benefits is that the mesh works as host formwork to receive shotcrete, which allows curvatures that could not be possible to obtain using a layer-by-layer extrusion process without formwork.

3.3. Fresh state requirements for 3D printed concrete

The fresh state properties of printed concrete are crucial to address its buildability, which consists of the ability to obtain the desirable shape with minimal deformation, avoiding collapse. Fresh state printed concrete presents a viscous behavior and it is usually assumed as a visco-plastic Bingham material that flows when the stresses surpass its yield stress [82]. According to several authors [64,79,82,83], the properties that are relevant for determining the printability of concrete ma-

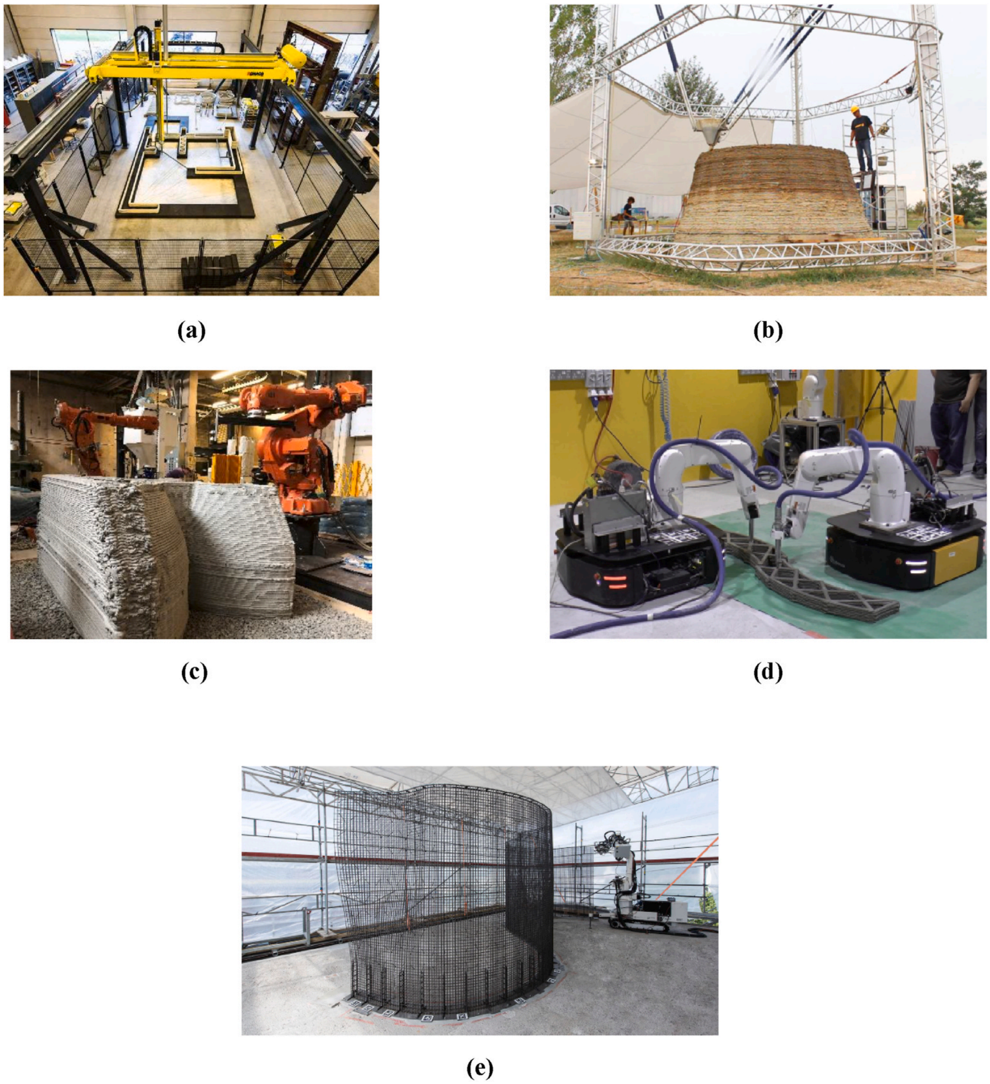


Fig. 18. Concrete extrusion-based 3D printing systems: (a) gantry system, Eindhoven University of Technology, Netherlands [79]; (b) delta system, WASP [80]; (c) stationary robotic arm system, The Pennsylvania State University; (d) system of small cooperative mobile robots, Nanyang Technological University, Singapore [78]; and (e) mesh mold system, ETH Zurich & DFAB House, Switzerland [81].

terials are: (i) rheology; (ii) stress-strain behavior; and (iii) time-dependent behavior. These properties can be summed into two main failure criteria:

- Strength-based failure criterion: based on yield stress. An adequate yield stress is crucial to assure stability after deposition of the printed layer, to resist the gravity load from the layers above.
- Stability-based (buckling) criterion: based on maximum printable height, which depends on Young's modulus. Buckling failure is related to successive lateral deformations and second order effects.

The determination of stiffness and yield stress is required to develop models that respectively predict failure by buckling or plastic collapse, which are validated by comparing results from finite element analysis with printing experiments [61,64,84]. Stiffness is tested and monitored on the basis of deformation, which depends on Young's modulus and is quantified by either stress-strain relation using the unconfined compression test for cylindrical specimens (ASTM D4648), optical metrology [85] or through PZT (piezoelectrical transducers) patches that assess stiffness gain and damage evolution, thereby ensuring the stability of the stacking process [86]. Time-dependent yield stress depends on the cohesiveness of the cement paste, which is suggested by several authors [64,65,87] to resemble the behavior of a cohesive soil, as failure in compression results from relative movement of particles [83]. In fact, its strength is attributed to a combination of inter particle friction and cohesion, which can be translated by a Mohr-Coulomb yield criterion, similarly to cohesive soils: $\tau_y = C(t) + \sigma_n \tan(\phi(t))$, where τ_y is the yield stress, C corresponds to the cohesion between particles, σ_n is the normal stress, and ϕ is the internal friction angle between particles. The static yield stress evolution over time model is presented in Fig. 19. Reflocculation (R_{thix}) and structuration (A_{thix}) correspond respectively to the gradient of the yield stress in the short (2 min) and long terms. An alternative way to perform yield stress analysis is through the dynamic stress sweep method using an oscillatory rheometer, which allows proper reproducibility of results and minimizes wall slippage problems in high-viscosity mortars [88,89].

3.4. Summary of main challenges

Additive manufacturing of concrete is still in its initial stage. Material properties of both fresh and hardened state concrete are under research and fabrication methods are still being developed and refined, while strategies to transition from small prototypes and computer models to large scale structures still need to be developed. The main chal-

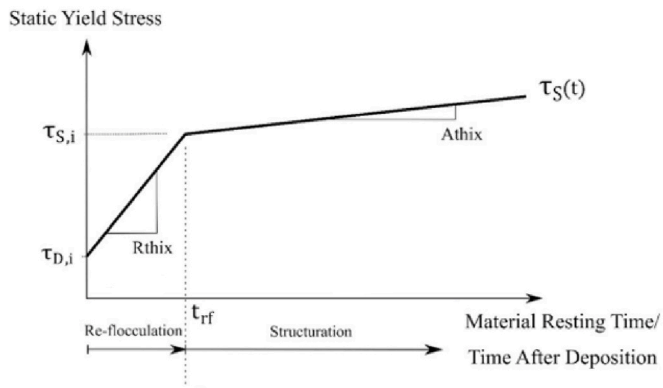


Fig. 19. Static yield shear stress evolution as a function of resting time [83].

lenges regarding this transition involve a combination of current material and process limitations. The production of non-supported structures, such as slabs and overhanging elements stands out as one of the main problems since it is highly dependent on the fresh state behavior of the material. In fact, as construction moves toward 3d printing of concrete, geometries that could be obtained by traditional means need to be reevaluated for the material and process restraints, since low yield strength of early age concrete limits the geometry of planar structures, including overhanging elements [90]. Another issue concerns tensile strength requirements of the material, which can be enhanced by incorporating reinforcement. However, while automated processes and strategies to reinforce 3d printed concrete are undergoing development, a way to surpass the lack of tensile strength can be accomplished by optimizing the design of the structure to maximize its reliance on compressive behavior. In addition, as the scale and complexity of structures increase, research is required to develop appropriate toolpath generation strategies. As far as the printing process is concerned, the transition to large scale construction brings additional difficulties. Gantry and delta systems need to be larger than the structure, whereas robotic arms solutions might involve large operational costs due to acquisition of several robots and complex operations to move them around the construction site. Besides scaling problems, there are operational issues caused by concrete setting in the extruding machines and piping.

4. Strategies for 3D printing houses and shelters

4.1. Main considerations

This section aims at identifying appropriate designs for unsupported 3D printing of concrete structures under compression. From the historical survey, it was possible to observe a prevalence of systems consisting of bearing walls toppled by shell roof structures, such as vaults and domes. Several printable designs are thus possible:

- Pure vault structures, where the structure consists exclusively of the vault;
- Bearing walls with vaulted roof;
- Pure dome structure, where the structure consists exclusively of the dome;
- Bearing walls with a pointed dome, which may need elements to transition between the rectangular shape of the walls and the round shape of the dome;
- Hybrid solutions, combining bearing walls with several vaulted and/or dome roof shapes.

In addition, the design should include solutions for:

- Foundations.
- Openings, including doors and windows.
- Location and placement of mechanical and electrical installations.

The making of vaults and domes by 3D printings relies on the deposition of successive layers, which can be done following the radial (or Roman), horizontal (or Corbeling), or diagonal (or Nubian) layering processes mentioned in section 2.6. with the nozzle positioned respectively tangent to the curve, or perpendicular to the layers, or as a combination of both.

4.2. Wall system

Including wall elements in the design increase the ceiling's height and provide a basis for placing the roofing system. From a structural viewpoint, walls have the function of bearing gravity loads from the roof. Two types of walls can be obtained in additive construction: (i) solid walls; (ii) hollow walls; and (iii) solid walls with openings - built on self-supporting overhangs. To include windows or doors, the wall needs to have discontinuities, which can be created with self-supporting overhangs, forming an angle larger than 65° – the maximum printing angle [42,91] (Fig. 20a and b). In the case of walls obtained from an extrusion process, besides analyzing shear strength failure, it is crucial to evaluate failure due to buckling, which is a problem since the number of deposited layers tends to increase without thickening the wall, leading to instability and further collapse.

4.3. Vault elements

As mentioned above, a bearing wall combined with a vault allows a viable and efficient compressive system to sustain deadloads. The process to construct vaulted structures with 3D printed concrete differs from traditional brick and stone masonry systems. Below, we present and discuss some possibilities for 3d printed vaults.

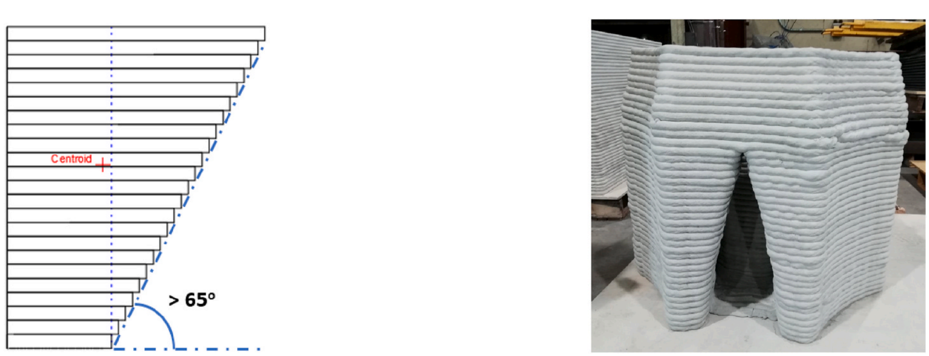


Fig. 20. Printed maximum overhang: (a) front view of layer deposition with maximum overhang of 65° and (b) self-supporting overhang [92].

4.3.1. Pointed arch vault

A viable approach to 3d print a pointed arch vault is to use the Nubian technique, which permits the construction of the vault without formwork by printing inclined layers of material against each other. The layers are planar and obtained by intersecting the vault with inclined parallel planes, in which the nozzle is perpendicular to the correspondent layers, as demonstrated by several studies [59,93]. An alternative method to obtain a pointed arch vault is through a hybrid solution using Nubian vaulting from both ends, and then infilling the middle region with a mix of Corbeling and Nubian vaulting, where a combination of horizontal and inclined layers is employed. Both approaches are diagrammed in Fig. 21.

4.3.2. Pointed cross-vault

When the goal is to have windows on all sides, the cross vault provides an adequate solution. The major difficulty is the cantilevered structures formed at each of the four corners during printing, which need to be analyzed in terms of displacement and tensile forces to guarantee they do not collapse. Also, to ensure stability, the projection of the center of mass of each cantilever during construction phase should fall within its base, which can be addressed by the use of counterweights at the base of each cantilever. Therefore, the adoption of a pointed, less curved arch as the base element of this geometry can increase its printability, but with the downside of increasing the height of the vault and creating a non-flat top, (Fig. 22).

As far as printing strategies are concerned, the first strategy would consist of corbeling the full structure, which can be feasible by adopting an adequate curvature and placing counterweights at the base of each cantilever, to ensure stability during printing. An alternative strategy is a hybrid solution in which a Nubian layering process closes the vault, in a squinch fashion, after joining the four corners through corbeling (Fig. 23).

4.3.3. Cloister-vault

In the case of the cloister dome, two similar methods can be adopted to materialize the desired form. The first method is pure corbeling and the second is a hybrid solution consisting of corbeling and further Nubian vaulting to enclose the final part of the dome (Fig. 24), which requires an inclined nozzle. In cloister vaults, the length of layer beads decrease as upper layers are printed, which decreases the time for the lower layers to harden and may cause premature collapse of the structure. This difficulty can be overcome by slowing down the printing speed when approaching the top, so that the material in the lower layers have time to harden.

4.3.4. Fan vault

The fan vault is another option available, in which the ribs of the vault present the same curvature and are rotated around the support axis. In the 3D printing of this vault, each layer would present a semicircular shape and be deposited on the top of each other, forming a slope that had to be larger than 65° avoid collapse [42]. The difficulty in obtaining this geometry stems from the need to use a corbelling process, where the layers are deposited with the nozzle in the vertical position. To guarantee the printability of the vault, the base element must be the pointed arch, which permits to guarantee that the maximum printing angle is not exceeded. Fig. 25 shows how a Fan vault system could be 3d printing in concrete. First, the side walls and the conoids are printed

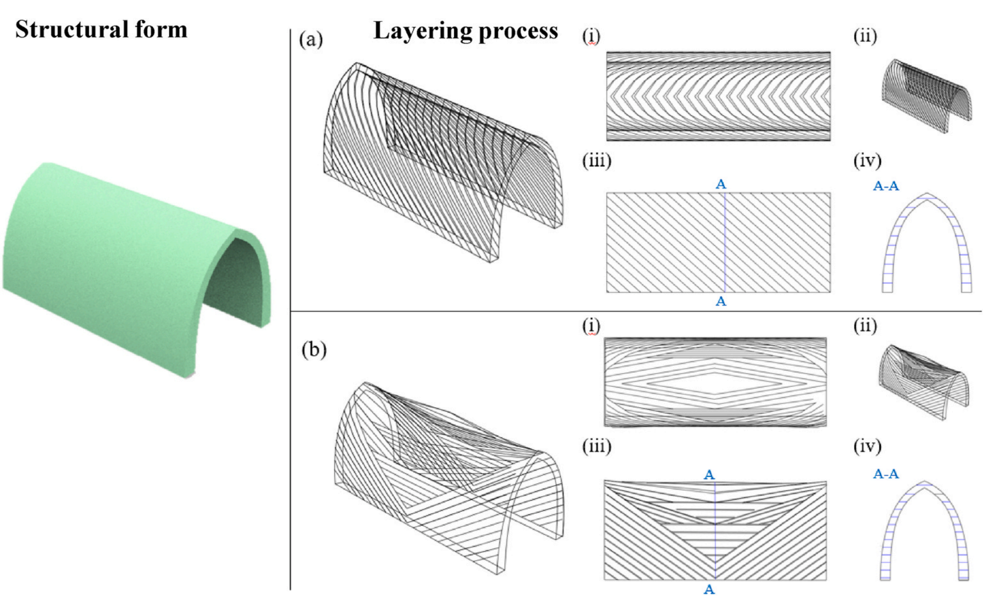


Fig. 21. Pointed arch vault printing: (a) pure Corbeling and (b) hybrid Corbeling and Nubian vault – (i) top view, (ii) perspective, (iii) left view, and (iv) right view.

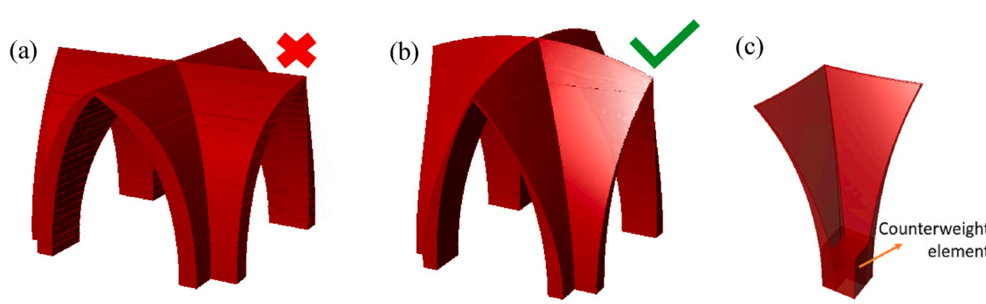


Fig. 22. Cross-vault design: (a) regular, (b) pointed, and (c) cantilever element.

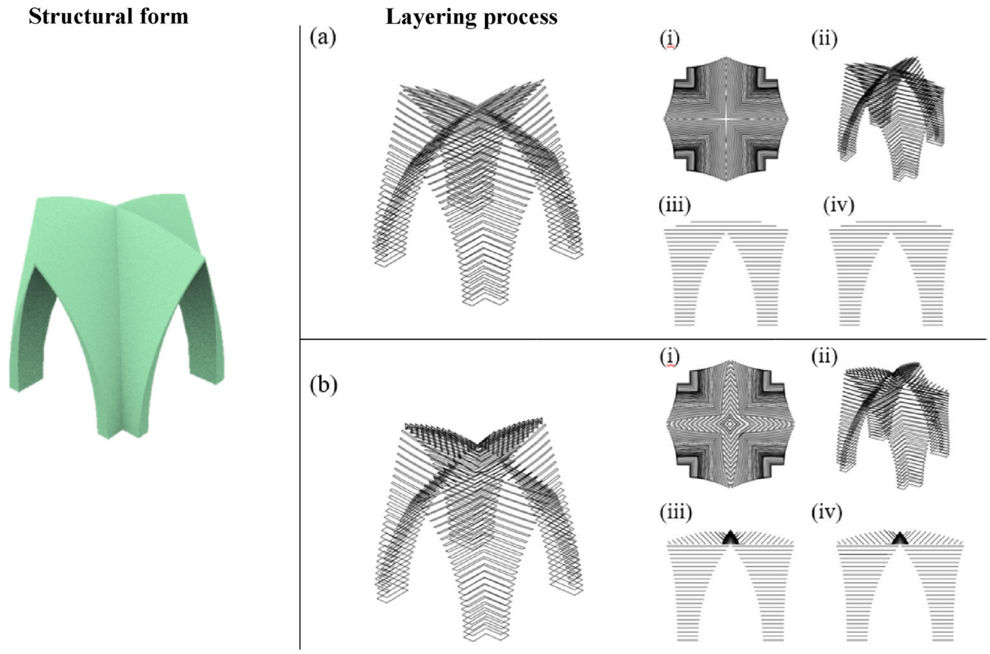


Fig. 23. Cross-vault printing: (a) pure Corbeling and (b) hybrid Corbeling and Nubian – (i) top view, (ii) perspective, (iii) left view, and (iv) right view.

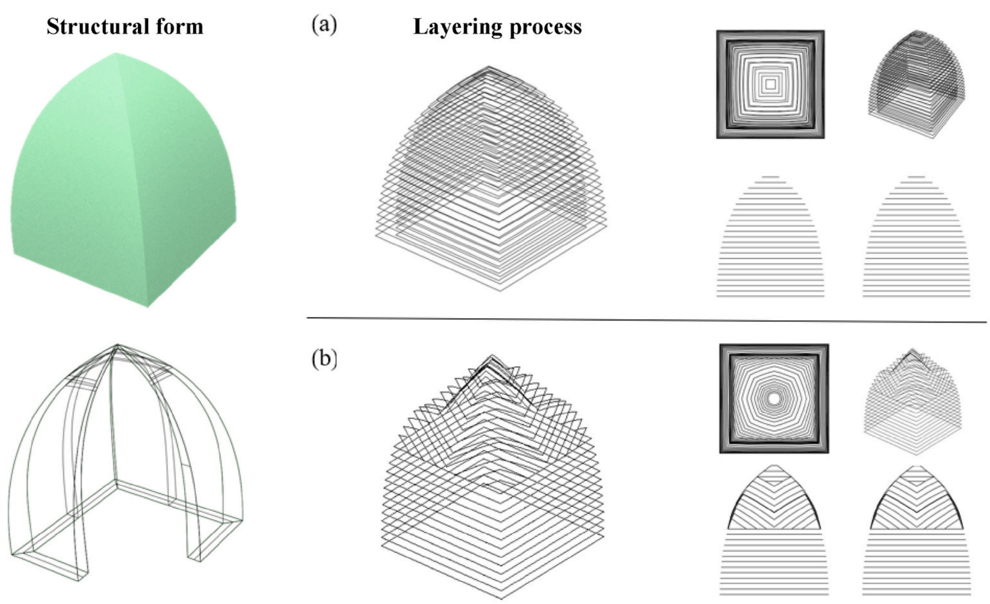


Fig. 24. Cloister-vault printing: (a) pure Corbeling and (b) hybrid Corbeling and Nubian – (i) top view, (ii) perspective, (iii) left view, and (iv) right view.

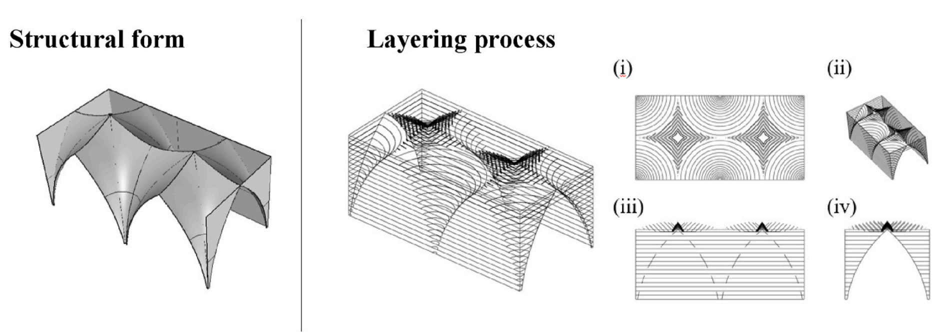


Fig. 25. Pointed dome printing: (a) pure Corbeling and (b) hybrid Corbeling and Nubian – (i) top view, (ii) perspective, (iii) left view, and (iv) right view.

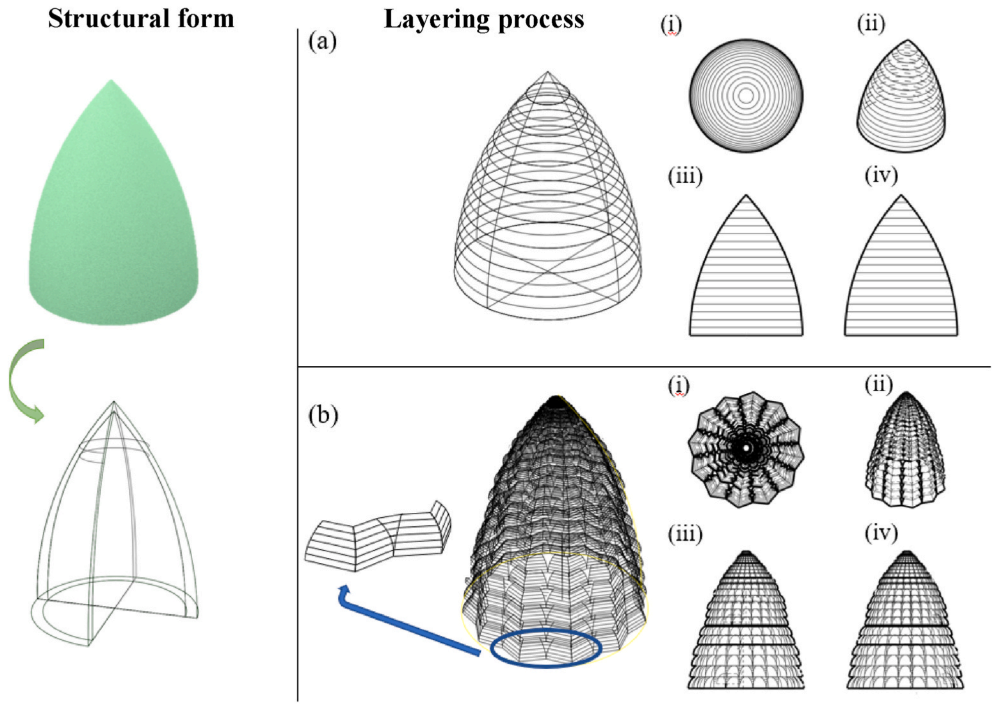


Fig. 26. Pointed dome printing: (a) pure corbeling and (b) hybrid Corbeling and Patkaneh – (i) top view, (ii) perspective, (iii) left view, and (iv) right view.

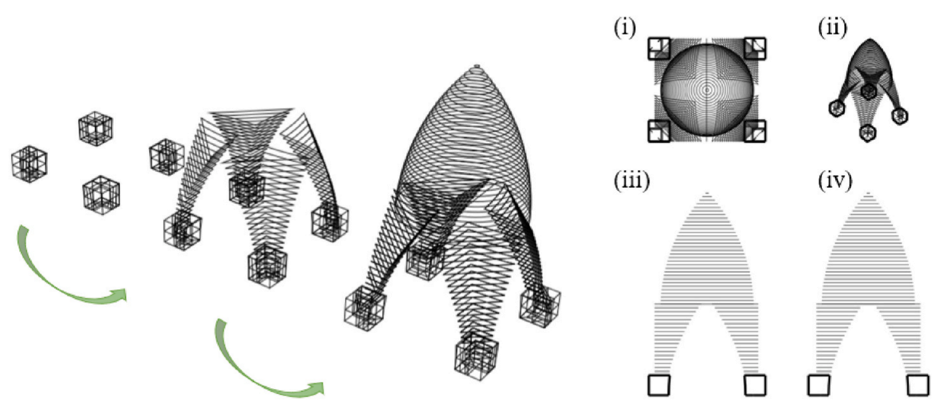


Fig. 27. Pointed dome on pendentives printing - (i) top view, (ii) perspective, (iii) left view, and (iv) right view.

through Corbeling and then, the spandrel, which encloses the vault, is built through Nubian vaulting, in a squinch fashion.

4.4. Dome elements

Material deposition of horizontal layers is one way to print dome elements. Since no formwork is used, a hemispherical dome is not an option due to lack of stability during printing. A pointed dome would be a more appropriate dome shape for unsupported 3D printing of concrete. If the dome is printed on top of an octagonal or hexadecagonal base, which approximates a circle, no transition elements are required, whereas if it is printed on a square base, transition elements such as pendentives and squinches are necessary.

4.4.1. Pointed dome

The simplest way to obtain a dome is through corbeling of layers, which is possible if the base arch is pointed with an adequate inclination (Fig. 26a). Besides the regular use of corbeling to deposit material in circular layers until the pointed dome is reached, a composition of niches, resembling *Patkaneh*, a type of Persian squinch, can be obtained using corbeling (Fig. 26b).

4.4.2. Transition elements – dome on pendentives and squinches

When the goal is to make a structure consisting of a dome supported by bearing walls, the introduction of squinches or pendentives, as in Persian architecture, can provide a way of reducing stress in the connections between the wall and the circular base of the dome, which will tend to open due to tensile hoop stresses resulting from deadloads. In addition, these transition elements permit to place the circular base of the dome on the top of the square base of the bearing wall. A process to obtain dome on pendentives is presented in Fig. 27. The main challenge underlying this structure is the pendentives, which consist of large cantilevers, thus presenting considerable displacements, and tensile forces on the outer face. In addition, an additional analysis concerning 2nd order effects should be performed, to account for incremental displacements during printing. A proper way to ease the printing of the pendentives is by adopting counterweights at the base of each pendentive, which slightly reduces its length and thus tensile forces. The printing of squinches can be accomplished by Corbelling, which makes use of successive cantilevers not exceeding the maximum printing angle, or by centered deposition where layers are printed with an inclined nozzle. The squinches also provide a way to absorb and direct the tensile forces from the base of the dome to the bearing walls. Two different methods

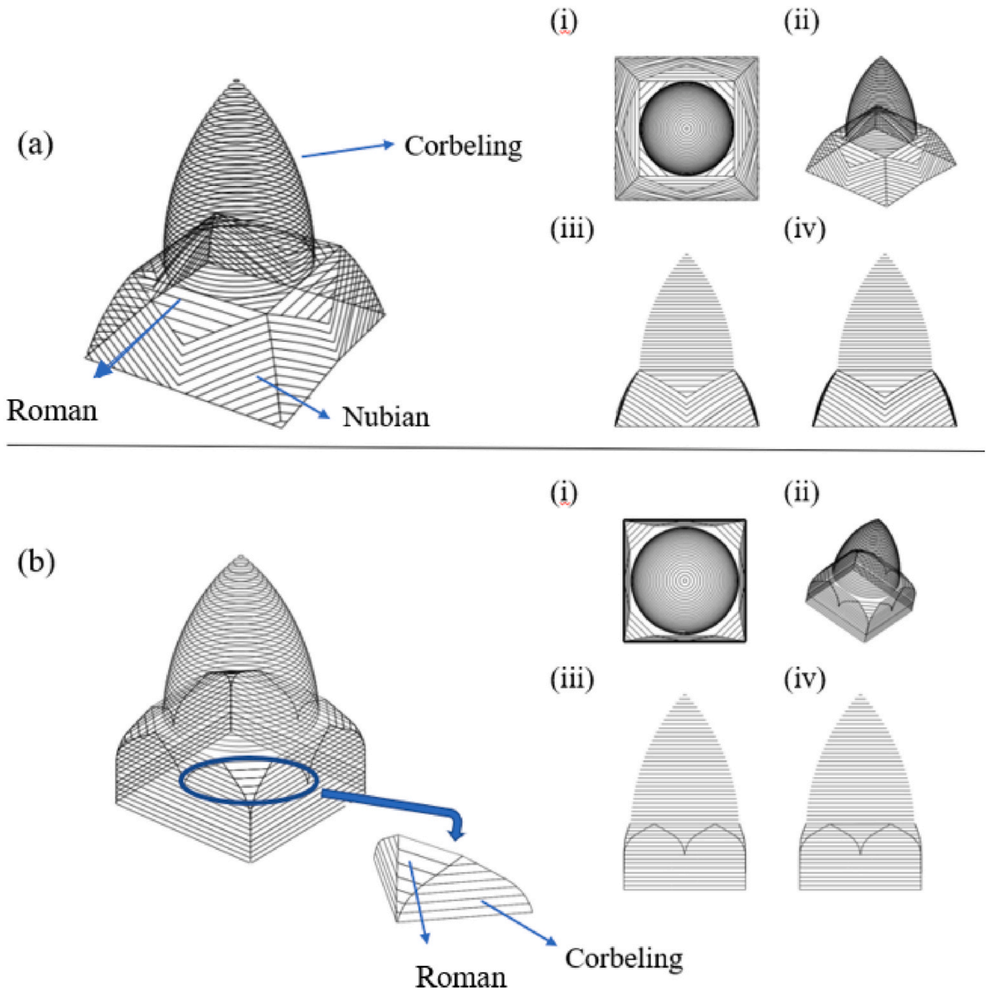


Fig. 28. Pointed dome on squinches printing: (a) hybrid Nubian and Corbeling, and (b) hybrid radial and Corbeling – (i) top view; (ii) perspective, (iii) left view, and (iv) right view.

can be used to make squinches. First, a variation of the Persian *Filpush* method, where successive inclined layers are extruded against each other starting from the corners (Fig. 28a), which consist of Catalan vaulting, with successive layers obtained from Roman vaulting to accommodate the circular base of the dome. The alternative consists of a smoother transition from the walls, printing squinches through Roman vaulting, requiring a radial layering of material (Fig. 28b), which requires the nozzle to be tangent to the curve of the vault.

5. Conclusions

The ubiquitous adoption of concrete 3D printing technology by the construction industry in the future will be highly dependent on advances in printing processes, materials science, reinforcement techniques, and design strategies. Nonetheless, the current state of the technology already permits the printing of structures with a reasonable scale that do not need formwork or reinforcement, which simplifies construction and reduces cost, making them suitable for affordable housing. To be effective, the geometry of these structures must be suited to the behavior of the printed material, turning them into compression-dominant structures. The analysis of historic precedents showed that building structures complying with such requirements combined load-bearing wall systems with shell structures, mostly obtained from arches, namely vaults and domes. The forms shown to be more promising from an unsupported 3D printing perspective are the barrel vault, cross vault, cloister vault, and pointed dome on squinches and pendentives. The Catalan vault and the fan vault also have some potential for 3D printing but pose greater challenges.

The printing feasibility of these structures, however, is highly dependent on their overall form, the fresh state properties of the material, on the ability to print with a non-vertical nozzle, and on toolpath design. The form will need to be optimized for the structural behavior during printing, when the shell form is incomplete, cantilevered parts may exist, and the material is still fresh; and after printing, when the shell structure has been completed, the material has hardened, and it is possible to take full advantage of the structure's compressive behavior. Optimization will determine the exact curvature and thickness of the shell in different areas. Regarding fresh properties, it is necessary to determine the strength of the printed material over time, to ensure it can endure the weight of subsequent layers and avoid collapse during printing. The analysis of historic precedents revealed three main strategies to create arched structures: Roman vaulting, Corbelling, and Nubian vaulting. In Corbelling and Nubian vaulting, the nozzle is perpendicular to the extruded layers and have already been tested, while Roman vaulting requires the nozzle to be tangent to the curve of the vault with extruded layers presenting a wedge section, which remains to be tested. Toolpath design involves breaking down the structure into parts that can be printed using one of the three vaulting methods above, which requires decomposing the overall form of the structure into smaller forms, then into layers and finally into beads. In this shape decomposition process, it will be necessary to design the length of the toolpath is such that previous layers of material will have time to harden enough before the next layer is deposited.

In summary, future research will be concerned with modelling some of these structures for structural analysis to assess their feasibility from a structural performance viewpoint and then print samples to test the printability and stability in fresh state.

Credit author statement

The paper discusses the possibility of printing in concrete historic structures that work mainly under compression. Gonçalo Duarte is the PhD student who carried out the research, advised by J. Duarte (main advisor) in design for additive manufacturing, and Ali Memari and N. Brown in concrete technology and structural analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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