**Due date for this assignment: 2022-06-01, 23:59 IST.**

**You may submit any number of times before the due date. The final submission will be considered for grading.**

1) Let $f : \mathbb{R} \to \mathbb{R}$ and $g : \mathbb{R} \to \mathbb{R}$ be two functions, defined as $f(x) = x^3 - 8x^2 + 7$ and $g(x) = -2f(x)$ respectively. Choose the correct option(s) from the following. ***2 points***

- ☐ $g$ has two turning points and there are no turning points with negative $y-$coordinate.
- ☐ $f$ has two turning points and there are no turning points with positive $y-$ coordinate.
- ☐ $f$ has two turning points and $y-$coordinate of only one turning point is positive.
- ☐ $g$ has two turning points and $y-$coordinate of only one turning point is negative.

2) Which among the following function first increases and then decreases in all the intervals $(-4, -3)$ and $(-1, 2)$ and $(5, 6)$. ***2 points***

- ☐ $\frac{1}{10000}(x+1)^2(x-2)(x+3)(x+4)(x-5)^2(x-6)^2$
- ☐ $\frac{1}{10000}(x+1)^2(x-2)(x+3)^2(x+4)(5-x)(x-6)^2$
- ☐ $\frac{1}{10000}(x+1)^2(x-2)(x+3)^2(x+4)^2(5-x)^2(x-6)^2(3-x)$
- ☐ $\frac{1}{10000}(x+1)^2(x-2)(x+3)(x+4)(x-5)^2(x-6)^2(x+7)$

3) Choose the set of correct options. ***3 points***

- ☐ There are infinitely many polynomial $p(x)$ of degree three such that $p(4) = 0, p(5) = 0, p(6) = 0$ and $p(0) = 1$.
- ☐ There is only one polynomial $p(x)$ of degree three such that $p(4) = 0, p(5) = 0, p(6) = 0$ and $p(0) = 1$.
- ☐ The number of turning points of $f(x) = (x-5)^8$ is 8.
- ☐ If $(x-1)$ is a factor of $2x^3 + x^2 + 7x + k$ then the value of $k$ is $-10$.
- ☐ The function $g(x) = -(x-2)(x+2)(x+3)(x+10)$ is positive in restricted domain $(\infty, -3)$.

4) Consider a polynomial function $p(x) = -(x^2-16)(x-3)^2(2-x)^2(x+9)$. Choose the set of correct options. ***2 points***

- ☐ $p(x)$ is strictly increasing when $x \in (-\infty, -9)$
- ☐ Total number of turning points of $p(x)$ are 7.
- ☐ Total number of turning points of $p(x)$ are 6.
- ☐ $p(x)$ is strictly decreasing when $x \in (4, \infty)$

5) Let $r(x)$ be a polynomial function which is obtained as the quotient after dividing the polynomial $p(x) = (x+5)(x-3)[x^2-4)$ by the polynomial $q(x) = (x-2)(2+x)$. Choose the correct option which represents the polynomial $r(x)$ most appropriately. ***2 points***

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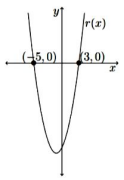


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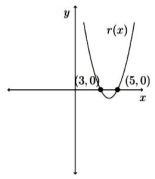


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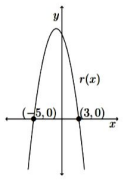


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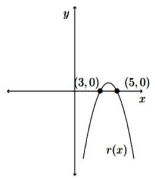


6) A function $f(x)$ which is the best fit for the data given in the Table-1 recorded by a student, is $f(x) = -(x-1)^2(x-3)(x-5)(x-7) + c$ What will be the value of $c$, so that SSE (Sum Squared Error) will be minimum?

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $y$ | 4 | 18 | 4 | −24 | 3 |

Table-1

***2 points***

7) An ant named $B$, wants to climb an uneven cliff and reach its anthill (i.e., home of ant). On its way home, $B$ makes sure that it collects some food. A group of ants have reached the food locations which are at $x-$intercepts of the function $f(x) = (x^2-30)\left((x-15)^3-1\right)$. As ants secrete pheromones (a form of signals which other ants can detect and reach the food location), $B$ gets to know the food location. Then the sum of the $x$-coordinates of all the food locations is

***1 point***

8) The Ministry of Road Transport and Highways wants to connect three aspirational districts with two roads $r_1$ and $r_2$. Two roads are connected if they intersect. The shape of the two roads $r_1$ and $r_2$ follows polynomial curve $f(x) = (x-8)(x-6)^2$ and $g(x) = -(x-8)(x-6)$ respectively. What will be the $x-$coordinate of the third aspirational district, if the first two are at $x-$intercepts of $f(x)$ and $g(x)$.

***3 points***

9) If $a$, $b$ and $c$ are the roots of the polynomial $3x^3 + 11x - 22$ and sum of the roots is 0, then find the value of $a^3 + b^3 + c^3$.

***3 points***

10) Let $f(x)$ and $g(x)$ be two functions defined from $\mathbb{R}$ to $\mathbb{R}$ such that

$$f(x) = \begin{cases} x^3 - 64 & \text{if } x \in (-\infty, -4) \\ x+1 & \text{if } x \in (-4, 0) \cup (0, 2) \cup [2, \infty) \\ 0 & \text{if } x = 0 \\ 1 & \text{if } x \in \{-4, 2\} \end{cases},$$

$g(x) = |x| - 1$

Find the value of $f(4)g(-4) + \frac{f(0)}{g(-6)} + f(3) - g(-39)$.

***1 point***

Submit Answers