Assignment 1

2MA201-Calculus and Differential Equations

Q 1. Test the convergence of the following

a. $\sum \frac{1}{\sqrt{n}+\sqrt{n+1}}$

b. $\sum_{n=1}^{\infty} \left(1-\frac{1}{n}\right)^{n^2}$

c. $\frac{x}{1.2}+\frac{x^2}{3.4}+\frac{x^3}{5.6}+\frac{x^4}{7.8}+\cdots, x>0$

d. $\sum_{n=0}^{\infty} \frac{n!}{e^n}$

e. $1+x+\frac{x^2}{2!}+\frac{x^3}{3!}++\frac{x^4}{4!}+\cdots, x>0$

f. $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n\sqrt{n}}$

g. $\frac{6}{3}-\frac{8}{5}+\frac{10}{7}-\frac{12}{9}+\cdots$

Q 2. Expand the function $\log\sec\left(x+\frac{\pi}{4}\right)$ in power of x and hence find $\log\sec\ 46^o$

Q. 3. Expand $\sin x$ in power of $\left(x+\frac{\pi}{6}\right)$

Q. 4 Expand $\log(x+h)$ in power of h.

Q. 5. If $u=\frac{(x+y)}{(y-z)}$, prove that $\frac{\partial^2 u}{\partial x \partial y}+\frac{\partial^2 u}{\partial y \partial z}+\frac{\partial^2 u}{\partial z^2}=\frac{\partial^2 u}{\partial x^2}$

Q. 6. If $f(x,y)=\frac{1}{\sqrt{y}}e^{-\frac{(x-a)^2}{4y}}$, show that $\frac{\partial^2 f}{\partial x^2}-\frac{\partial f}{\partial y}=0.$

Q. 7. If $\theta=t^{-3/2}e^{-r^2/4t}$, prove that $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2.\frac{\partial\theta}{\partial r}\right)=\frac{\partial\theta}{\partial t}$.

Q. 8. If $z=x^n f_1\left(\frac{y}{x}\right)+y^{-n} f_2\left(\frac{x}{y}\right)$, then show that:

$$x^2\frac{\partial^2 z}{\partial x^2}+2xy\frac{\partial^2 z}{\partial x \partial y}+y^2\frac{\partial^2 z}{\partial y^2}+x\frac{\partial z}{\partial x}+y\frac{\partial z}{\partial y}=n^2 z$$

Q. 9. If $u=f(e^{y-z},e^{z-x},e^{x-y})$, then show that $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0.$