

## TP1: Example of a recursive function

### Python Recursive Function

In Python, we know that a [function](#) can call other functions. It is even possible for the function to call itself. These types of construct are termed as recursive functions.

The following image shows the working of a recursive function called `recurse`.

```
def recurse():  
    ...  
    recurse()  
    ...  
recurse()
```



The diagram shows a Python function definition for `recurse()`. The function body contains three lines: an ellipsis (`...`), a call to `recurse()`, and another ellipsis (`...`). Below the function definition, there is a call to `recurse()`. A teal line originates from the `recurse()` call at the bottom, extends horizontally to the right, then vertically upwards, then horizontally to the left, and finally vertically downwards to point at the `recurse()` line within the function's body. The text "recursive call" is written vertically next to the upward and leftward segments of this line.

Recursive Function in Python

## TP1: Example of a recursive function

```
def factorial(x):  
    """This is a recursive function  
    to find the factorial of an integer"""  
  
    if x == 1:  
        return 1  
    else:  
        return (x * factorial(x-1))  
  
num = 3  
print("The factorial of", num, "is", factorial(num))
```

**Output:???**

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```

**Output:** The factorial of 3 is 6

## TP1: Example of a recursive function

### Example

Use recursion to add all of the numbers up to 10.

```
public class MyClass {
    public static void main(String[] args) {
        int result = sum(10);
        System.out.println(result);
    }
    public static int sum(int k) {
        if (k > 0) {
            return k + sum(k - 1);
        } else {
            return 0;
        }
    }
}
```

**Output:???**

## Output:

10 + sum(9)

10 + ( 9 + sum(8) )

10 + ( 9 + ( 8 + sum(7) ) )

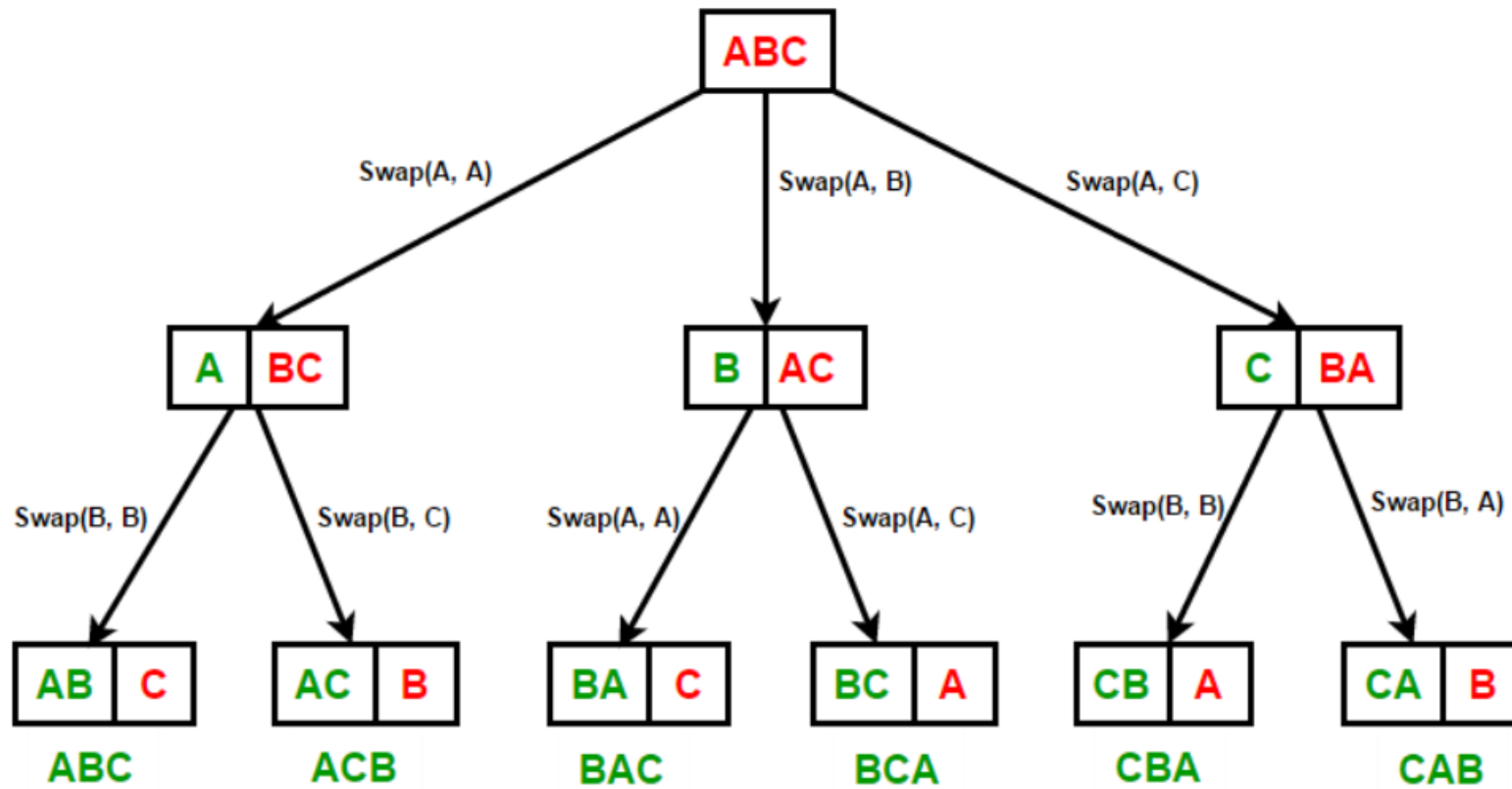
...

10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 + sum(0)

10 + 9 + 8 + 7 + 6 + 5 + 4 + 3 + 2 + 1 + 0

# Permutation of a string of unique character

Below is the recursion tree for printing all permutations of string "ABC".



Recursion Tree for string "ABC"

```
class Main
{
    // Utility function to swap two characters in a character array
    private static void swap(char[] ch, int i, int j)

    // Recursive function to generate all permutations of a String
    private static void permutations(char[] ch, int currentIndex)

    // generate all permutations of a String in Java
    public static void main(String[] args)

}
```

## Dynamic programming

**Memoization** is a technique for implementing dynamic programming to make recursive algorithms efficient. It often has the same benefits as regular dynamic programming without requiring major changes to the original more natural recursive algorithm.

### Basic Idea

- The first thing is to design the natural recursive algorithm.
- If recursive calls with the same arguments are repeatedly made, then the inefficient recursive algorithm can be memoized by saving these subproblem solutions in a table so they do not have to be recomputed.



## Implementation

To implement **memoization** to recursive algorithms, a table is maintained with subproblem solutions, but the control structure for filling in the table occurs during normal execution of the recursive algorithm. This can be summarized in steps:

1. A memoized recursive algorithm maintains an entry in a table for the solution to each of subproblem,
2. Each table entry initially contains a special value to indicate that entry has yet to be filled in.
3. When the subproblem is first encountered, its solution is computed and stored in the table.
4. Subsequently, the value is looked up rather than computed

To illustrate the steps above, let's take an example for computing nth Fibonacci number with a recursive algorithm as:

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To illustrate the steps above, let's take an example for computing nth Fibonacci number with a recursive algorithm as:

```
// without memoization
static int fib(int n) {
    if (n == 0 || n == 1) return n;
    return fib(n - 1) + fib(n - 2);
}
```

**What is the issue?**

## Maximum Product of Two Sequences Problem