Chapter 7

Algebraic Expressions

Algebraic expression can be defined as the combination of constants and variables connected by algebraic operations such as addition, subtraction, multiplication and division. In this expression, parts which are connected by these algebraic operations are known as terms of the expression.

Types of algebraic Expressions:

(1) Monomial: The expression which is having only one term is called a monomial. For example: 3x, -5abc etc.
(2) Binomial: The expression which is having two terms is known as binomial. For example: y + 3, 6 – 8x etc.
(3) Trinomial: The expression which is having three terms is known as trinomial. For example: $3x + 5y - 7$ etc.
(4) Quadrinomial: The expression which is having four terms is known as quadrinomial. For example: $x + y + z - 4$ etc
(5) Polynomial: The expression which is having two or more terms is known as polynomial.

Like terms: Those terms which are having the same factors are known as like terms.

Unlike terms: Those terms which are not having the same factors are known as unlike terms.

Procedure to find out the value of an Algebraic Expression:

1) Write the algebraic expression

2) Obtaining the values of literals (variables) involved.

3) Replacing each literal by its numerical value.

4) Simplifying the expression so obtained.

Examples:

Example 1 – Identify the monomials, binomials, trinomials and quadrinomials from the following expressions:

(i) $4x^2$

Solution: Since the given expression $4x^2$ contains only one term thus it is a monomial expression.

(ii) $x^2 - 1$

Solution: Since the given expression $x^2 - 1$ contains two terms thus it is a binomial expression.

(iii) $x^2 - y^2$

Solution: Since the given expression $x^2 - y^2$ contains two terms thus it is a binomial expression.

(iv) $3x^2 + 4y^2 + 5z$

Solution: Since the given expression $3x^2 + 4y^2 + 5z$ contains three terms thus it is a trinomial expression.

(v) $ax^2 + bx + c$

Solution: Since the given expression $ax^2 + bx + c$ contains three terms thus it is a trinomial expression.

(vi) $a^2 + b^2 + c^2 - d^2$

Solution: Since the given expression $a^2 + b^2 + c^2 - d^2$ contains four terms thus it is a quadrinomial expression.

(vii) $3ab^2$

Solution: Since the given expression $3ab^2$ contains only one term thus it is a monomial expression.

(viii) $a^3 + b^3 - 3ab + 5$

Solution: Since the given expression $a^3 + b^3 - 3ab + 5$ contains four terms thus it is a quadrinomial expression.

(ix) $-xyz$

Solution: Since the given expression $-xyz$ contains only one term thus it is a monomial expression.

(x) $3x - 2$

Solution: Since the given expression $3x - 2$ contains two terms thus it is a binomial expression.

(xi) $4x - 3x$

Solution: Since $4x - 3x = x$ which is a monomial as it contains only one term. Thus $4x - 3x$ is a monomial expression.

Example 2 – Write all the terms of each of the following algebraic expressions:

(i) $3x^5 + 5y^4 - 7x^2y + 7$

Solution: We can see that the given expression $3x^5 + 5y^4 - 7x^2y + 7$ contains four terms. The terms are as follows: $3x^5$, $5y^4$, $-7x^2y$ and 7

(ii) $9y^3 - 2z^3 + 7x^3y - 3xyz$

Solution: We can see that the given expression $9y^3 - 2z^3 + 7x^3y - 3xyz$ contains four terms. The terms are as follows: $9y^3$, $-2z^3$, $7x^3y$ and $-3xyz$

(iii) $a^5 - 3ab - b^2 + 6$

Solution: We can see that the given expression $a^5 - 3ab - b^2 + 6$ contains four terms. The terms are as follows: a^5, $-3ab$, $-b^2$ and 6

(iv) $x^2 - x + 1$

Solution: We can see that the given expression $x^2 - x + 1$ contains three terms. The terms are as follows: x^2, $-x$ and 1

Example 3 – Write down the coefficient of x in each of the following:

(i) $3x$

Solution: The coefficient of x in the given expression $3x$ is 3.

(ii) $-4ax$

Solution: The coefficient of x in the given expression $-4ax$ is $-4a$

(iii) $5xy^2$

Solution: The coefficient of x in the given expression $5xy^2$ is $5y^2$

(iv) xyz

Solution: The coefficient of x in the given expression xyz is yz

(v) $-\frac{3}{2}x + 5$

Solution: The coefficient of x in the given expression $-\frac{3}{2}x + 5$ is $-\frac{3}{2}$

(vi) $-\frac{5}{2}xyz^2$

Solution: The coefficient of x in the given expression $-\frac{5}{2}xyz^2$ is $-\frac{5}{2}yz^2$

Example 4 – Write the numerical coefficient of each term of the following algebraic expressions:

(i) $x^2 - 7x^2y + 5xy^2 - 2$

Solution: We can see that the given expression $x^2 - 7x^2y + 5xy^2 - 2$contains four terms. The terms are x^2, $-7x^2y$, $5xy^2$ and -2 and the corresponding numerical coefficients of terms are 1, -7, 5 and -2 respectively.

(ii) $-2a^3 + 7ab^2 - 6ab + 8$

Solution: We can see that the given expression $-2a^3 + 7ab^2 - 6ab + 8$ contains four terms. The terms are $-2a^3, 7ab^2, -6ab$ and 8 and the corresponding numerical coefficients of terms are -2, 7, -6 and 8 respectively.

Example 5 – Identify the like terms in each of the following:

(i) x^2, y^2, $2x^2$, z^2

Solution: Here, x^2 and $2x^2$ are like terms as both have the same literal factor x^2.

(ii) $2xy$, yz, $3x$, $\frac{yz}{2}$

Solution: Here, yz and $\frac{yz}{2}$ are like terms as both have the same literal factor yz.

(iii) $-2x^2y$, x^2z, $-yx^2$, x^2y^2

Solution: Here, $-2x^2y$ and $-yx^2$ are like terms as both have the same literal factor x^2y.

(iv) cab^2, a^2bc, b^2ac, c^2ab, ab^2c, abc, acb^2

Solution: Here, cab^2, b^2ac, ab^2c and acb^2 are like terms as all are having the same literal factor b^2ac.

Example 6 – Identify the like terms in each of the following algebraic expressions:

(i) $x - 2y + 3z - 4x + 3xy$

Solution: Here, x and $-4x$ are like terms as both have the same literal factor x.

(ii) $3a + 2b - c + \frac{3}{2}a - 4 + 3b$

Solution: Here, $3a$ and $-\frac{3}{2}a$ are like terms as both have the same literal factor a. Also, $2b$ and $3b$ are like terms as both have the same literal factor b.

(iii) $xy^2 + 3x^2y - 4x^2y^2 - 5y^2x - 2z^2x + 3xz^2$

Solution: Here, xy^2 and $-5y^2x$ are like terms as both have the same literal factor xy^2. Also, $-2z^2x$ and $3xz^2$ are like terms as both have the same literal factor xz^2.

Example 7 – Evaluate each of the following algebraic expressions for x = 2, y = - 3, z = - 2, a = 2, b = 3:

(i) $2a^2 + ab$

Solution: We will substitute the value of a = 2 and b = 3 in the given expression as follows:

$2a^2 + ab = 2(2)^2 + (2 \times 3)$

$= (2 \times 4) + (2 \times 3) = 8 + 6 = 14$

(ii) $2a^2 + x^2 - y^2$

Solution: We will substitute the value of a = 2, x = 2 and y = -3 in the given expression as follows:

$2a^2 + x^2 - y^2 = 2(2)^2 + (2)^2 - (-3)^2$

$= (2 \times 4) + (2 \times 2) - (-3 \times -3)$

$= 8 + 4 - 9 = 12 - 9 = 3$

(iii) $x^3 - y^3 + z^3$

Solution: We will substitute the value of x = 2, y = -3 and z = -2 in the given expression as follows:

$x^3 - y^3 + z^3 = (2)^3 - (-3)^3 + (-2)^3$

$= (2 \times 2 \times 2) - (-3 \times -3 \times -3) + (-2 \times -2 \times -2)$

$= 8 - (-27) + (-8) = 8 + 27 - 8 = 35 - 8 = 27$

(iv) $4xy^2 - 3yz^2 + 4x^2z$

Solution: We will substitute the value of x = 2, y = -3 and z = -2 in the given expression as follows:

$4xy^2 - 3yz^2 + 4x^2z = 4 \times 2 \times (-3)^2 - 3 \times (-3) \times (-2)^2 + 4 \times (2)^2 \times (-2)$

$= (4 \times 2 \times 9) - (3 \times -3 \times 4) + (4 \times 4 \times -2)$

$= 72 - (-36) + (-32) = 72 + 36 - 32 = 108 - 32 = 76$

(v) $x^3 + y^3 + 3xyz + ab$

Solution: We will substitute the value of x = 2, y = -3, z = -2, a = 2 and b = 3 in the given expression as follows:

$x^3 + y^3 + 3xyz + ab = (2)^3 + (-3)^3 + 3 \times 2 \times (-3) \times (-2) + 2 \times 3$

$= (2 \times 2 \times 2) + (-3 \times -3 \times -3) + (3 \times 2 \times -3 \times -2) + 2 \times 3$

$= 8 + (-27) + (36) + 6 = 8 - 27 + 36 + 6 = 8 + 36 + 6 - 27 = 50 - 27 = 23$

(vi) $5 + 4z^3 - 6y + 7a + xy$

Solution: We will substitute the value of x = 2, y = -3, z = -2 and a = 2 in the given expression as follows:

$5 + 4z^3 - 6y + 7a + xy = \ 5 + 4 \times (-2)^3 - 6 \times (-3) + 7 \times 2 + 2 \times (-3)$

$= 5 + (4 \times -2 \times -2 \times -2) - (6 \times -3) + (7 \times 2) + 2 \times -3$

$= 5 + (-32) - (-18) + 14 + (-6) = 5 - 32 + 18 + 14 - 6 = 5 + 18 + 14 - 32 - 6 = 37 - 38 = -1$

Exercise 7.1

Question 1 – Identify the monomials, binomials, trinomials and quadrinomials from the following expressions:

(i) a^2

Solution: Since the given expression a^2 contains only one term thus it is a monomial expression.

(ii) $a^2 - b^2$

Solution: Since the given expression $a^2 - b^2$ contains two terms thus it is a binomial expression.

(iii) $x^3 + y^3 + z^3$

Solution: Since the given expression $x^3 + y^3 + z^3$ contains three terms thus it is a trinomial expression.

(iv) $x^3 + y^3 + z^3 + 3xyz$

Solution: Since the given expression $x^3 + y^3 + z^3 + 3xyz$ contains four terms thus it is a quadrinomial expression.

(v) $7 + 5$

Solution: Since $7 + 5 = 12$ which is a monomial containing only one term. Thus $7 + 5$ is a monomial expression.

(vi) $abc + 1$

Solution: Since the given expression $abc + 1$ contains two terms thus it is a binomial expression.

(vii) $3x - 6 + 5$

Solution: Since $3x - 6 + 5 = 3x - 1$ which is a binomial containing two terms. Thus $3x - 6 + 5$ is a binomial expression.

(viii) $2x - 3y + 4$

Solution: Since the given expression $2x - 3y + 4$ contains three terms thus it is a trinomial expression.

(ix) $xy + yz + zx$

Solution: Since the given expression $xy + yz + zx$ contains three terms thus it is a trinomial expression.

(x) $ax^3 + bx^2 + cx + d$

Solution: Since the given expression $ax^3 + bx^2 + cx + d$ contains four terms thus it is a quadrinomial expression.

Question 2 – Write all the terms of each of the following algebraic expressions:

(i) $3x$

Solution: We can see that the given expression $3x$ contains only one term. The term is $3x$

(ii) $2x - 3$

Solution: We can see that the given expression $2x - 3$ contains two terms. The terms are as follows: $2x$ and -3

(iii) $2x^2 - 7$

Solution: We can see that the given expression $2x^2 - 7$ contains two terms. The terms are as follows: $2x^2$ and -7

(iv) $2x^2 + y^2 - 3xy + 4$

Solution: We can see that the given expression $2x^2 + y^2 - 3xy + 4$ contains four terms. The terms are as follows: $2x^2$, y^2, $-3xy$ and 4

Question 3 – Identify the like terms and also mention the numerical coefficients of those terms:

(i) $4xy, -5x^2y, -3yx, 2xy^2$

Solution: Here, $4xy$ and $-3yx$ are like terms as both have the same literal factor xy and the numerical coefficients are 4 and -3 respectively.

(ii) $7a^2bc, -3ca^2b, -\frac{5}{2}abc^2, \frac{3}{2}abc^2, -\frac{4}{3}cba^2$

Solution: Here, $7a^2bc$, $-3ca^2b$ and $-\frac{4}{3}cba^2$ are like terms as both have the same literal factor a^2bc and the numerical coefficients are 7, -3 and $-\frac{4}{3}$ respectively. Also, $-\frac{5}{2}abc^2$ and $\frac{3}{2}abc^2$ are like terms having the same literal factor c^2ab and the numerical coefficient are $-\frac{5}{2}$ and $\frac{3}{2}$ respectively.

Question 4 – Identify the like terms in the following algebraic expressions:

(i) $a^2 + b^2 - 2a^2 + c^2 + 4a$

Solution: Here, a^2 and $-2a^2$ are like terms as both have the same literal factor a^2

(ii) $3x + 4xy - 2yz + \frac{5}{2}zy$

Solution: Here, $-2yz$ and $\frac{5}{2}zy$ are like terms as both have the same literal factor yz

(iii) $abc + ab^2c + 2acb^2 + 3c^2ab + b^2ac - 2a^2bc + 3cab^2$

Solution: Here, b^2c, $2acb^2$, b^2ac and $3cab^2$ are like terms as all have the same literal factor a^2bc.

Question 5 – Write the coefficient of x in the following:

(i) $-12x$

Solution: The coefficient of x in the given expression $-12x$ is -12

(ii) $-7xy$

Solution: The coefficient of x in the given expression $-7xy$ is $-7y$

(iii) xyz

Solution: The coefficient of x in the given expression xyz is yz

(iv) $-7ax$

Solution: The coefficient of x in the given expression $-7ax$ is $-7a$

Question 6 – Write the coefficient of x^2 in the following:

(i) $-3x^2$

Solution: The coefficient of x^2 in the given expression $-3x^2$ is -3

(ii) $5x^2yz$

Solution: The coefficient of x^2 in the given expression $5x^2yz$ is $5yz$

(iii) $\frac{5}{7}x^2z$

Solution: The coefficient of x^2 in the given expression $\frac{5}{7}x^2z$ is $\frac{5}{7}z$

(iv) $\frac{-3}{2}ax^2 + yx$

Solution: The coefficient of x^2 in the given expression $\frac{-3}{2}ax^2 + yx$ is $\frac{-3}{2}a$

Question 7 – Write the coefficient of:

(i) $y\ in\ -3y$

Solution: The coefficient of y in the given expression $-3y$ is -3

(ii) $a\ in\ 2ab$

Solution: The coefficient of a in the given expression $2ab$ is $2b$

(iii) $z\ in\ -7xyz$

Solution: The coefficient of z in the given expression $-7xyz$ is $-7xy$

(iv) $p\ in\ -3pqr$

Solution: The coefficient of p in the given expression $-3pqr$ is $-3qr$

(v) $y^2\ in\ 9xy^2z$

Solution: The coefficient of y^2 in the given expression $9xy^2z$ is $9xz$

(vi) $x^3\ in\ x^3 + 1$

Solution: The coefficient of x^3 in the given expression $x^3 + 1$ is 1

(vii) $x^2\ in\ -x^2$

Solution: The coefficient of x^2 in the given expression $-x^2$ is -1

Question 8 – Write the numerical coefficient of each of the following:

(i) xy

Solution: We can see that the given expression xy contains one term. The term is xy and the corresponding numerical coefficient of term is 1.

(ii) $-6yz$

Solution: We can see that the given expression $-6yz$ contains one term. The term is $-6yz$ and the corresponding numerical coefficient of term is -6.

(iii) $7abc$

Solution: We can see that the given expression $7abc$ contains one term. The term is $7abc$ and the corresponding numerical coefficient of term is 7.

(iv) $-2x^3y^2z$

Solution: We can see that the given expression $-2x^3y^2z$ contains one term. The term is $-2x^3y^2z$ and the corresponding numerical coefficient of term is -2.

Question 9 – Write the numerical coefficient of each term in the following algebraic expressions:

(i) $4x^2y - \frac{3}{2}xy + \frac{5}{2}xy^2$

Solution: We can see that the given expression $4x^2y - \frac{3}{2}xy + \frac{5}{2}xy^2$ contains three terms. The terms are $4x^2y$, $-\frac{3}{2}xy$ and $\frac{5}{2}xy^2$ and the corresponding numerical coefficients of terms are 4, $-\frac{3}{2}$ and $\frac{5}{2}$ respectively.

(ii) $-\frac{5}{3}x^2y + \frac{7}{4}xyz + 3$

Solution: We can see that the given expression $-\frac{5}{3}x^2y + \frac{7}{4}xyz + 3$ contains three terms. The terms are $-\frac{5}{3}x^2y$, $\frac{7}{4}xyz$ and 3 and the corresponding numerical coefficients of terms are $-\frac{5}{3}$, $\frac{7}{4}$ and 3 respectively.

Question 10 – Write the constant term of each of the following algebraic expressions:

(i) $x^2y - xy^2 + 7xy - 3$

Solution: In the given expression, -3 is the constant term as it does not contain any literal factor

(ii) $a^3 - 3a^2 + 7a + 5$

Solution: In the given expression, 5 is the constant term as it does not contain any literal factor

Question 11 – Evaluate each of the following expressions for $x = -2$, $y = -1$, $z = 3$:

(i) $\frac{x}{y} + \frac{y}{z} + \frac{z}{x}$

Solution: We will substitute the value of x = -2, y = -1 and z = 3 in the given expression as follows:

$\frac{x}{y} + \frac{y}{z} + \frac{z}{x} = \frac{-2}{-1} + \frac{-1}{3} + \frac{3}{-2}$

$= 2 - \frac{1}{3} - \frac{3}{2}$

$= \frac{12-2-9}{6} = \frac{12-11}{6} = \frac{1}{6}$

(ii) $x^2 + y^2 + z^2 - xy - yz - zx$

Solution: We will substitute the value of x = -2, y = -1and z = 3 in the given expression as follows:

$x^2 + y^2 + z^2 - xy - yz - zx = (-2)^2 + (-1)^2 + 3^2 - (-2) \times (-1) - (-1) \times (3) - (3) \times (-2)$

$= (-2 \times -2) + (-1 \times -1) + (3 \times 3) - 2 - (-3) - (-6)$

$= 4 + 1 + 9 - 2 + 3 + 6$

$= 21$

Question 12 – Evaluate each of the following algebraic expressions for $x = 1$, $y = -1$, $z = 2$, $a = -2$, $b = 1$, $c = -2$:

(i) $ax + by + cz$

Solution: We will substitute the value of $x = 1, y = -1, z = 2, a = -2, b = 1\ and\ c = -2$ in the given expression as follows:

$ax + by + cz = (-2) \times (1) + 1 \times (-1) + (-2) \times (2)$

$= -2 - 1 - 4 = -7$

(ii) $ax^2 + by^2 - cz^2$

Solution: We will substitute the value of $x = 1, y = -1, z = 2, a = -2, b = 1 \; and \; c = -2$ in the given expression as follows:

$ax^2 + by^2 - cz^2 = (-2) \times (1)^2 + 1 \times (-1)^2 - (-2) \times (2)^2$

$= (-2) \times 1 + 1 \times 1 - (-2) \times 4$

$= -2 + 1 - (-8)$

$= -2 + 1 + 8 = 7$

(iii) $axy + byz + cxy$

Solution: We will substitute the value of $x = 1, y = -1, z = 2, a = -2, b = 1 \; and \; c = -2$ in the given expression as follows:

$axy + byz + cxz = (-2) \times (1) \times (-1) + 1 \times (-1) \times (2) + (-2) \times (1) \times (-1)$

$= 2 + (-2) + 2$

$= 2 - 2 + 2 = 4 - 2 = 2$

Operations on Algebraic Expressions

(1) Addition: There are two methods in order to add the algebraic expressions.

(a) Horizontal method: According to this method, we write all the expressions in a horizontal line and then arrange the like terms together and then add the terms.

(b) Column method: According to this method, we write each expression in a separate row which are to be arranged in a manner that like terms in a row are below the other in another row.

(2) Subtraction: While subtracting two expressions by column method, we indicate the change of signs of every term in the expression to be subtracted below the original sign of each term.

Examples:

Example 1 – Add the following:

(i) $3x + 2y$ *and* $x + y$

Solution: Using horizontal method, we have

$(3x + 2y) + (x + y)$

$= (3x + x) + (2y + y)$

$= 4x + 3y$

(ii) $x + y + 3$ *and* $3x + 2y + 5$

Solution: Using column method, we have

$$\begin{array}{r} x + y + 3 \\ + \quad 3x + 2y + 5 \\ \hline 4x + 3y + 8 \end{array}$$

(iii) $2x + 3y + z$ *and* $2x - y - z$

Solution: Using column method, we have

$$\begin{array}{r} 2x + 3y + z \\ + \quad 2x - y - z \\ \hline 4x + 2y \end{array}$$

Example 2 – Add:

(i) $xy^2 + 4x^2y - 7x^2y - 3xy^2 + 3$ *and* $x^2y + xy^2$

Solution: Using horizontal method, we have

$(xy^2 + 4x^2y - 7x^2y - 3xy^2 + 3) + (x^2y + xy^2)$

Now, we collect like terms in separate brackets and solve as follows:

$= (xy^2 - 3xy^2 + xy^2) + (4x^2y - 7x^2y + x^2y) + 3$

$= xy^2(1 - 3 + 1) + x^2y(4 - 7 + 1) + 3$

$= xy^2(-1) + x^2y(-2) + 3$

$= -xy^2 - 2x^2y + 3$

(ii) $5x^2 + 7y - 6z^2$, $4y + 3x^2$, $9x^2 + 2z^2 - 9y$ *and* $2y - 2x^2$

Solution: Using horizontal method, we have

$(5x^2 + 7y - 6z^2) + (4y + 3x^2) + (9x^2 + 2z^2 - 9y) + (2y - 2x^2)$

Now, we collect like terms in separate brackets and solve as follows:

$= (5x^2 + 3x^2 + 9x^2 - 2x^2) + (7y + 4y - 9y + 2y)) + (-6z^2 + 2z^2)$

$= x^2(5 + 3 + 9 - 2) + y(7 + 4 - 9 + 2) + z^2(-6 + 2)$

$= x^2(15) + y(4) + z^2(-4)$

$= 15x^2 + 4y - 4z^2$

Example 3 – Subtract:

(i) $5x$ *from* $9x$

Solution: Using horizontal method, we have

$9x - 5x = x(9 - 5) = 4x$

(ii) $-7x$ *from* $5x$

Solution: Using horizontal method, we have

$5x - (-7x) = 5x + 7x = x(5 + 7) = 12x$

(iii) $-8a$ *from* $-3a$

Solution: Using horizontal method, we have

$(-3a) - (-8a) = -3a + 8a = a(-3 + 8) = 5a$

Example 4 – Subtract: $a^2 - 3ab$ *from* $2a^2 - 7ab$

Solution: Using horizontal method, we have

$(2a^2 - 7ab) - (a^2 - 3ab)$

We will change the signs from '-'to '+' and from '+' to '-'in the second bracket as follows:

$2a^2 - 7ab - a^2 + 3ab$

Now, we collect like terms in separate brackets and solve as follows:

$= (2a^2 - a^2) + (-7ab + 3ab)$

$= a^2(2 - 1) + ab(-7 + 3)$

$= a^2(1) + ab(-4)$

$= a^2 + (-4)ab$

$= a^2 - 4ab$

Example 5 – Subtract: $x^2 - 3xy + 7y^2 - 2$ *from* $6xy - 4x^2 - y^2 + 5$

Solution: Using horizontal method, we have

$(6xy - 4x^2 - y^2 + 5) - (x^2 - 3xy + 7y^2 - 2)$

We will change the signs from '-'to '+' and from '+' to '-'in the second bracket as follows:

$6xy - 4x^2 - y^2 + 5 - x^2 + 3xy - 7y^2 + 2$

Now, we collect like terms in separate brackets and solve as follows:

$= (6xy + 3xy) + (-4x^2 - x^2) + (-y^2 - 7y^2) + (5 + 2)$

$= xy(6 + 3) + x^2(-4 - 1) + y^2(-1 - 7) + 7$

$= xy(9) + x^2(-5) + y^2(-8) + 7$

$= 9xy - 5x^2 - 8y^2 + 7$

Example 6 – From the sum of $4x^4 - 3x^3 + 6x^2$, $4x^3 + 4x - 3$ *and* $-3x^4 - 5x^2 + 2x$ subtract $5x^4 - 7x^3 - 3x + 4$

Solution: Firstly we will solve $(4x^4 - 3x^3 + 6x^2) + (4x^3 + 4x - 3) + (-3x^4 - 5x^2 + 2x)$ by using horizontal method as follows:

We collect like terms in separate brackets and solve as follows:

$= (4x^4 - 3x^4) + (-3x^3 + 4x^3) + (6x^2 - 5x^2) + (4x + 2x) + (-3)$

$= x^4(4 - 3) + x^3(-3 + 4) + x^2(6 - 5) + x(4 + 2) - 3$

$= x^4(1) + x^3(1) + x^2(1) + x(6) - 3$

$= x^4 + x^3 + x^2 + 6x - 3$

Now, we subtract $5x^4 - 7x^3 - 3x + 4$ from $x^4 + x^3 + x^2 + 6x - 3$ using horizontal method as follows:

$(x^4 + x^3 + x^2 + 6x - 3) - (5x^4 - 7x^3 - 3x + 4)$

We will change the signs from '-'to '+' and from '+' to '-'in the second bracket as follows:

$x^4 + x^3 + x^2 + 6x - 3 - 5x^4 + 7x^3 + 3x - 4$

Now, we collect like terms in separate brackets and solve as follows:

$(x^4 - 5x^4) + (x^3 + 7x^3) + x^2 + (6x + 3x) + (-3 - 4)$

$= (1-5)x^4 + (1+7)x^3 + x^2 + (6+3)x + (-7)$

$= -4x^4 + 8x^3 + x^2 + 9x - 7$

Example 7 – What should be added to $a^2 + 2ab + b^2$ to obtain $4ab + b^2$

Solution: Let the required number be 'X'

Then according to given question, we have

$(a^2 + 2ab + b^2) + X = 4ab + b^2$

$\Rightarrow X = (4ab + b^2) - (a^2 + 2ab + b^2)$

Now, we will solve by using horizontal method

We will change the signs from '-'to '+' and from '+' to '-'in the second bracket as follows:

$\Rightarrow X = 4ab + b^2 - a^2 - 2ab - b^2$

$\Rightarrow X = (4ab - 2ab) + (b^2 - b^2) - a^2$

$\Rightarrow X = 2ab - a^2$

Thus, the required number is $2ab - a^2$

Example 8 – What should be subtracted from $a^3 - 4a^2 + 5a - 6$ to obtain $a^2 - 2a + 1$?

Solution: Let the required number be 'X'

Then according to given question, we have

$(a^3 - 4a^2 + 5a - 6) - X = a^2 - 2a + 1$

$\Rightarrow X = (a^3 - 4a^2 + 5a - 6) - (a^2 - 2a + 1)$

Now, we will solve by using horizontal method

We will change the signs from '-'to '+' and from '+' to '-'in the second bracket as follows:

$\Rightarrow X = a^3 - 4a^2 + 5a - 6 - a^2 + 2a - 1$

=> $X = a^3 + (-4a^2 - a^2) + (5a + 2a) + (-6 - 1)$

=> $X = a^3 + (-5a^2) + (7a) + (-7)$

=> $X = a^3 - 5a^2 + 7a - 7$

Thus, the required number is $a^3 - 5a^2 + 7a - 7$

Example 9 – How much is $x^3 - 2x^2 + x + 4$ greater than $2x^3 + 7x^2 - 5x + 6$?

Solution: We will solve the following: $(x^3 - 2x^2 + x + 4) - (2x^3 + 7x^2 - 5x + 6)$

We will change the signs from '-'to '+' and from '+' to '-'in the second bracket as follows:

$x^3 - 2x^2 + x + 4 - 2x^3 - 7x^2 + 5x - 6$

$= (x^3 - 2x^3) + (-2x^2 - 7x^2) + (x + 5x) + (4 - 6)$

$= -x^3 - 9x^2 + 6x + (-2)$

$= -x^3 - 9x^2 + 6x - 2$

Example 10 – How much is $2a^2 - 7a + 5$ less than $a^3 - 3a^2 + 2a - 3$?

Solution: We will solve the following: $(a^3 - 3a^2 + 2a - 3) - (2a^2 - 7a + 5)$

We will change the signs from '-'to '+' and from '+' to '-'in the second bracket as follows:

$a^3 - 3a^2 + 2a - 3 - 2a^2 + 7a - 5$

$= (a^3) + (-3a^2 - 2a^2) + (2a + 7a) + (-3 - 5)$

$= a^3 + (-5a^2) + 9a + (-8)$

$= a^3 - 5a^2 + 9a - 8$

Example 11 – How much does $2a^2 - 5a + 4$ exceed $3a^3 - 5a^2 + 7a - 9$?

Solution: We will solve the following: $(2a^2 - 5a + 4) - (3a^3 - 5a^2 + 7a - 9)$

We will change the signs from '-'to '+' and from '+' to '-'in the second bracket as follows:

$2a^2 - 5a + 4 - 3a^3 + 5a^2 - 7a + 9$

= $(-3a^3) + (2a^2 + 5a^2) + (-5a - 7a) + (4 + 9)$

= $-3a^3 + 7a^2 + (-12a) + 13$

= $-3a^3 + 7a^2 - 12a + 13$

Exercise 7.2

Question 1 – Add the following:

(i) $3x$ *and* $7x$

Solution: Using horizontal method, we have

$(3x) + (7x)$

= $(3 + 7)x$

= $10x$

(ii) $-5xy$ *and* $9xy$

Solution: Using horizontal method, we have

$(-5xy) + (9xy)$

= $(-5 + 9)xy$

= $4xy$

Question 2 – simplify each of the following:

(i) $7x^3y + 9yx^3$

Solution: Using horizontal method, we have

$7x^3y + 9yx^3 = (7x^3y) + (9x^3y)$

= $(7 + 9)x^3y$

= $16x^3y$

(ii) $12a^2b + 3ba^2$

Solution: Using horizontal method, we have

$(12a^2b) + (3ba^2) = 12a^2b + 3a^2b$

$= (12 + 3)a^2b$

$= 15a^2b$

Question 3 – Add the following:

(i) $7abc, -5abc, 9abc, -8abc$

Solution: Using horizontal method, we have

$7abc + (-5abc) + 9abc + (-8abc)$

$= (7 - 5 + 9 - 8)abc$

$= (16 - 13)abc$

$= 3abc$

(ii) $2x^2y, -4x^2y, 6x^2y, -5x^2y$

Solution: Using horizontal method, we have

$2x^2y + (-4x^2y) + 6x^2y + (-5x^2y)$

$= (2 - 4 + 6 - 5)x^2y$

$= (8 - 9)x^2y$

$= -x^2y$

Question 4 – Add the following expressions:

(i) $x^3 - 2x^2y + 3xy^2 - y^3$, $2x^3 - 5xy^2 + 3x^2y - 4y^3$

Solution: Using horizontal method, we have

$(x^3 - 2x^2y + 3xy^2 - y^3) + (2x^3 - 5xy^2 + 3x^2y - 4y^3)$

Now, we collect like terms in separate brackets and solve as follows:

$= (x^3 + 2x^3) + (-2x^2y + 3x^2y) + (3xy^2 - 5xy^2) + (-y^3 - 4y^3)$

$= (1 + 2)x^3 + (-2 + 3)x^2y + (3 - 5)xy^2 + (-1 - 4)y^3$

$= 3x^3 + x^2y + (-2)xy^2 + (-5)y^3$

$= 3x^3 + x^2y - 2xy^2 - 5y^3$

(ii) $a^4 - 2a^3b + 3ab^3 + 4a^2b^2 + 3b^4$, $-2a^4 - 5ab^3 + 7a^3b - 6a^2b^2 + b^4$

Solution: Using horizontal method, we have

$(a^4 - 2a^3b + 3ab^3 + 4a^2b^2 + 3b^4) + (-2a^4 - 5ab^3 + 7a^3b - 6a^2b^2 + b^4)$

Now, we collect like terms in separate brackets and solve as follows:

$= (a^4 - 2a^4) + (-2a^3b + 7a^3b) + (3ab^3 - 5ab^3) + (4a^2b^2 - 6a^2b^2) + (3b^4 + b^4)$

$= (1 - 2)a^4 + (-2 + 7)a^3b + (3 - 5)ab^3 + (4 - 6)a^2b^2 + (3 + 1)b^4$

$= -a^4 + 5a^3b + (-2)ab^3 + (-2)a^2b^2 + 4b^4$

$= -a^4 + 5a^3b - 2ab^3 - 2a^2b^2 + 4b^4$

Question 5 – add the following expressions:

(i) $8a - 6ab + 5b$, $-6a - ab - 8b$ and $-4a + 2ab + 3b$

Solution: Using horizontal method, we have

$(8a - 6ab + 5b) + (-6a - ab - 8b) + (-4a + 2ab + 3b)$

Now, we collect like terms in separate brackets and solve as follows:

$= (8a - 6a - 4a) + (-6ab - ab + 2ab) + (5b - 8b + 3b)$

$= (8 - 6 - 4)a + (-6 - 1 + 2)ab + (5 - 8 + 3)b$

$= -2a + (-5)ab + 0b$

$= -2a - 5ab$

(ii) $5x^3 + 7 + 6x - 5x^2$, $2x^2 - 8 - 9x$, $4x - 2x^2 + 3x^3$, $3x^3 - 9x - x^2$, $x - x^2 - x^3 - 4$

Solution: Using horizontal method, we have

$(5x^3 + 7 + 6x - 5x^2) + (2x^2 - 8 - 9x) + (4x - 2x^2 + 3x^3) + (3x^3 - 9x - x^2) + (x - x^2 - x^3 - 4)$

Now, we collect like terms in separate brackets and solve as follows:

$= (5x^3 + 3x^3 + 3x^3 - x^3) + (-5x^2 + 2x^2 - 2x^2 - x^2 - x^2) + (6x - 9x + 4x - 9x + x) + (7 - 8 - 4)$

$= (5 + 3 + 3 - 1)x^3 + (-5 + 2 - 2 - 1 - 1)x^2 + (6 - 9 + 4 - 9 + 1)x + (7 - 8 - 4)$

$= 10x^3 + (-7)x^2 + (-7)x + (-5)$

$= 10x^3 - 7x^2 - 7x - 5$

Question 6 – Add the following:

(i) $x - 3y - 2z$

$5x + 7y - 8z$

$3x - 2y + 5z$

Solution: Using column method, we have

$$\begin{array}{r} x - 3y - 2z \\ + \quad 5x + 7y - 8z \\ \hline 3x - 2y + 5z \\ \hline 9x + 2y - 5z \\ \hline \end{array}$$

(ii) $4ab - 5bc + 7ca$

$-3ab + 2bc - 3ca$

$5ab - 3bc + 4ca$

Solution: Using column method, we have

$$\begin{array}{r} 4ab - 5bc + 7ca \\ + \quad -3ab + 2bc - 3ca \\ \hline 5ab - 3bc + 4ca \\ \hline 6ab - 6bc + 8ca \\ \hline \end{array}$$

Question 7 – Add $2x^2 - 3x + 1$ to the sum of $3x^2 - 2x$ and $3x + 7$

Solution: Firstly we will solve $(3x^2 - 2x) + (3x + 7)$ by using horizontal method

We collect like terms in separate brackets and solve as follows:

$= (3x^2) + (-2x + 3x) + 7$

$= 3x^2 + x + 7$

Now, we will solve $(2x^2 - 3x + 1) + (3x^2 + x + 7)$

$= (2x^2 + 3x^2) + (-3x + x) + (1 + 7)$

$= 5x^2 + (-2x) + 8$

$= 5x^2 - 2x + 8$

Question 8 – Add $x^2 + 2xy + y^2$ to the sum of $x^2 - 3y^2$ and $2x^2 - y^2 + 9$

Solution: Firstly we will solve $(x^2 - 3y^2) + (2x^2 - y^2 + 9)$ by using horizontal method

We collect like terms in separate brackets and solve as follows:

$= (x^2 + 2x^2) + (-3y^2 - y^2) + 9$

$= 3x^2 + (-4y^2) + 9 = 3x^2 - 4y^2 + 9$

Now, we will solve $(x^2 + 2xy + y^2) + (3x^2 - 4y^2 + 7)$

$= (x^2 + 3x^2) + 2xy + (y^2 - 4y^2) + 9$

$= 4x^2 + 2xy + (-3y^2) + 9$

$= 4x^2 + 2xy - 3y^2 + 9$

Question 9: Add $a^3 + b^3 - 3$ to the sum of $2a^3 - 3b^3 - 3ab + 7$ and $-a^3 + b^3 + 3ab - 9$

Solution: Firstly we will solve $(2a^3 - 3b^3 - 3ab + 7) + (-a^3 + b^3 + 3ab - 9)$ by using horizontal method

We collect like terms in separate brackets and solve as follows:

$= (2a^3 - a^3) + (-3b^3 + b^3) + (-3ab + 3ab) + (7 - 9)$

$= a^3 + (-2b^3) + 0 + (-2) = a^3 - 2b^3 - 2$

Now, we will solve $(a^3 + b^3 - 3) + (a^3 - 2b^3 - 2)$

$= (a^3 + a^3) + (b^3 - 2b^3) + (-3 - 2)$

$= 2a^3 + (-b^3) + (-5)$

$= 2a^3 - b^3 - 5$

Question 10 – Subtract:

(i) $7a^2b$ *from* $3a^2b$

Solution: Using horizontal method, we have

$(3a^2b) - (7a^2b) = (3 - 7)a^2b = -4a^2b$

(ii) $4xy$ *from* $-3xy$

Solution: Using horizontal method, we have

$(-3xy) - (4xy) = (-3 - 4)xy = -7xy$

Question 11 – Subtract:

(i) – $4x$ *from* $3y$

Solution: Using horizontal method, we have

$(3y) - (-4x) = 3y + 4x$

(ii) – $2x$ *from* $-5y$

Solution: Using horizontal method, we have

$(-5y) - (-2x) = -5y + 2x$

Question 12 – Subtract:

(i) $6x^3 - 7x^2 + 5x - 3$ *from* $4 - 5x + 6x^2 - 8x^3$

Solution: Using horizontal method, we have

$(4 - 5x + 6x^2 - 8x^3) - (6x^3 - 7x^2 + 5x - 3)$

We will change the signs from '-'to '+' and from '+' to '-'in the second bracket as follows:

$4 - 5x + 6x^2 - 8x^3 - 6x^3 + 7x^2 - 5x + 3$

Now, we collect like terms in separate brackets and solve as follows:

$= (-8x^3 - 6x^3) + (6x^2 + 7x^2) + (-5x - 5x) + (4 + 3)$

$= (-8 - 6)x^3 + (6 + 7)x^2 + (-5 - 5)x + 7$

$= -14x^3 + 13x^2 + (-10)x + 7$

$= -14x^3 + 13x^2 - 10x + 7$

(ii) $-x^2 - 3z$ *from* $5x^2 - y + z + 7$

Solution: Using horizontal method, we have

$(5x^2 - y + z + 7) - (-x^2 - 3z)$

We will change the signs from '-'to '+' and from '+' to '-'in the second bracket as follows:

$5x^2 - y + z + 7 + x^2 + 3z$

Now, we collect like terms in separate brackets and solve as follows:

$= (5x^2 + x^2) - y + (z + 3z) + 7$

$= (5 + 1)x^2 - y + (1 + 3)z + 7$

$= 6x^2 - y + 4z + 7$

(iii) $x^3 + 2x^2y + 6xy^2 - y^3$ *from* $y^3 - 3xy^2 - 4x^2y$

Solution: Using horizontal method, we have

$(y^3 - 3xy^2 - 4x^2y) - (x^3 + 2x^2y + 6xy^2 - y^3)$

We will change the signs from '-'to '+' and from '+' to '-'in the second bracket as follows:

$y^3 - 3xy^2 - 4x^2y - x^3 - 2x^2y - 6xy^2 + y^3$

Now, we collect like terms in separate brackets and solve as follows:

$= (y^3 + y^3) + (-3xy^2 - 6xy^2) + (-4x^2y - 2x^2y) + (-x^3)$

$= (1 + 1)y^3 + (-3 - 6)xy^2 + (-4 - 2)x^2y + (-x^3)$

$= 2y^3 + (-9)xy^2 + (-6)x^2y - x^3$

$= 2y^3 - 9xy^2 - 6x^2y - x^3$

Question 13 – From

(i) $p^3 - 4 + 3p^2$, take away $5p^2 - 3p^3 + p - 6$

Solution: We will solve: $(p^3 - 4 + 3p^2) - (5p^2 - 3p^3 + p - 6)$

We will change the signs from '-'to '+' and from '+' to '-'in the second bracket as follows:

$p^3 - 4 + 3p^2 - 5p^2 + 3p^3 - p + 6$

Now, we collect like terms in separate brackets and solve as follows:

$= (p^3 + 3p^3) + (3p^2 - 5p^2) + (-p) + (-4 + 6)$

$= (1 + 3)p^3 + (3 - 5)p^2 + (-p) + 2$

$= 4p^3 + (-2)p^2 + (-p) + 2$

$= 4p^3 - 2p^2 - p + 2$

(ii) $7 + x - x^2$, take away $9 + x + 3x^2 + 7x^3$

Solution: We will solve: $(7 + x - x^2) - (9 + x + 3x^2 + 7x^3)$

We will change the signs from '-'to '+' and from '+' to '-'in the second bracket as follows:

$7 + x - x^2 - 9 - x - 3x^2 - 7x^3$

Now, we collect like terms in separate brackets and solve as follows:

$= (-7x^3) + (-x^2 - 3x^2) + (x - x) + (7 - 9)$

$= -7x^3 + (-1 - 3)x^2 + (-2)$

$= -7x^3 + (-4)x^2 + (-2)$

$= -7x^3 - 4x^2 - 2$

(iii)$1 - 5y^2$, take away $y^3 + 7y^2 + y + 1$

Solution: We will solve: $(1 - 5y^2) - (y^3 + 7y^2 + y + 1)$

We will change the signs from '-'to '+' and from '+' to '-'in the second bracket as follows:

$1 - 5y^2 - y^3 - 7y^2 - y - 1$

Now, we collect like terms in separate brackets and solve as follows:

$= (-y^3) + (-5y^2 - 7y^2) + (-y) + (1 - 1)$

$= -y^3 + (-5 - 7)y^2 + (-y)$

$= -y^3 + (-12)y^2 + (-y)$

$= -y^3 - 12y^2 - y$

(iv) $x^3 - 5x^2 + 3x + 1$, take away $6x^2 - 4x^3 + 5 + 3x$

Solution: We will solve: $(x^3 - 5x^2 + 3x + 1) - (6x^2 - 4x^3 + 5 + 3x)$

We will change the signs from '-'to '+' and from '+' to '-'in the second bracket as follows:

$x^3 - 5x^2 + 3x + 1 - 6x^2 + 4x^3 - 5 - 3x$

Now, we collect like terms in separate brackets and solve as follows:

$= (x^3 + 4x^3) + (-5x^2 - 6x^2) + (3x - 3x) + (1 - 5)$

$= (1 + 4)x^3 + (-5 - 6)x^2 + (-4)$

$= 5x^3 + (-11)x^2 + (-4)$

$= 5x^3 - 11x^2 - 4$

Question 14 – From the sum of $3x^2 - 5x + 2$ *and* $-5x^2 - 8x + 9$ subtract $4x^2 - 7x + 9$

Solution: Firstly we will solve $(3x^2 - 5x + 2) + (-5x^2 - 8x + 9)$ by using horizontal method as follows:

We collect like terms in separate brackets and solve as follows:

$= (3x^2 - 5x^2) + (-5x - 8x) + (2 + 9)$

$= (3 - 5)x^2 + (-5 - 8)x + 11$

$= -2x^2 + (-13)x + 11$

$= -2x^2 - 13x + 11$

Now, we subtract $4x^2 - 7x + 9$ from $(-2x^2 - 13x + 11)$ using horizontal method as follows:

$(-2x^2 - 13x + 11\ ) - (4x^2 - 7x + 9)$

We will change the signs from '-'to '+' and from '+' to '-'in the second bracket as follows:

$-2x^2 - 13x + 11\ \ -4x^2 + 7x - 9$

Now, we collect like terms in separate brackets and solve as follows:

$(-2x^2 - 4x^2) + (-13x + 7x) + (11 - 9)$

$= (-2 - 4)x^2 + (-13 + 7)x + 2$

$= -6x^2 + (-6)x + 2$

$= -6x^2 - 6x + 2$

Question 15 – Subtract the sum of $13x - 4y + 7z$ *and* $-6z + 6x + 3y$ from the sum of $6x - 4y - 4z$ *and* $2x + 4y - 7$

Solution: Firstly we will solve $(13x - 4y + 7z\ ) + (-6z + 6x + 3y)$ by using horizontal method as follows:

We collect like terms in separate brackets and solve as follows:

$= (13x + 6x) + (-4y + 3y) + (7z - 6z)$

$= (13 + 6)x + (-4 + 3)y + (7 - 6)z$

$= 19x + (-1)y + z$

$= 19x - y + z$

Now, we will solve $(6x - 4y - 4z) + (2x + 4y - 7)$ by using horizontal method as follows:

We collect like terms in separate brackets and solve as follows:

$= (6x + 2x) + (-4y + 4y) + (-4z) - 7$

$= (6 + 2)x + (-4 + 4)y + (-4)z - 7$

$= 8x - 4z - 7$

Now, we will subtract $(19x - y + z)$ from $(8x - 4z - 7)$ as follows:

$\Rightarrow (8x - 4z - 7) - (19x - y + z)$

We will change the signs from '-'to '+' and from '+' to '-'in the second bracket as follows:

$8x - 4z - 7 \ -19x + y - z$

Now, we collect like terms in separate brackets and solve as follows:

$(8x - 19x) + (y) + (-4z - z) - 7$

$= (8 - 19)x + y + (-4 - 1)z - 7$

$= -11x + y\ (-5)z - 7$

$= -11x + y - 5z - 7$

Question 16 – From the sum of $x^2 + 3y^2 - 6xy$, $2x^2 - y^2 + 8xy$ and $y^2 + 8$ and $x^2 - 3xy$ subtract $-3x^2 + 4y^2 - xy + x - y + 3$

Solution: Firstly we will solve $(x^2 + 3y^2 - 6xy) + (2x^2 - y^2 + 8xy) + (y^2 + 8) + (x^2 - 3xy)$ by using horizontal method as follows:

We collect like terms in separate brackets and solve as follows:

$= (x^2 + 2x^2 + x^2) + (3y^2 - y^2 + y^2) + (-6xy + 8xy - 3xy) + 8$

$= (1 + 2 + 1)x^2 + (3 - 1 + 1)y^2 + (-6 + 8 - 3)xy + 8$

$= 4x^2 + 3y^2 + (-1)xy + 8$

$= 4x^2 + 3y^2 - xy + 8$

Now, we subtract $-3x^2 + 4y^2 - xy + x - y + 3$ from $4x^2 + 3y^2 - xy + 8$ using horizontal method as follows:

$(4x^2 + 3y^2 - xy + 8) - (-3x^2 + 4y^2 - xy + x - y + 3)$

We will change the signs from '-'to '+' and from '+' to '-'in the second bracket as follows:

$4x^2 + 3y^2 - xy + 8 + 3x^2 - 4y^2 + xy - x + y - 3$

Now, we collect like terms in separate brackets and solve as follows:

$(4x^2 + 3x^2) + (3y^2 - 4y^2) + (-xy + xy) - x + y + (8 - 3)$

$= (4 + 3)x^2 + (3 - 4)y^2 - x + y + 5$

$= 7x^2 + (-1)y^2 - x + y + 5$

$= 7x^2 - y^2 - x + y + 5$

Question 17 – What should be added to $xy - 3yz + 4zx$ to get $4xy - 3zx + 4yz + 7$?

Solution: Let the required number be 'A'

Then according to given question, we have

$(xy - 3yz + 4zx) + A = 4xy - 3zx + 4yz + 7$

$\Rightarrow A = (4xy - 3zx + 4yz + 7) - (xy - 3yz + 4zx)$

Now, we will solve by using horizontal method

We will change the signs from '-'to '+' and from '+' to '-'in the second bracket as follows:

=> A $= 4xy - 3zx + 4yz + 7 - xy + 3yz - 4zx$

=> A $= (4xy - xy) + (-3zx - 4zx) + (4yz + 3yz) + 7$

=> $A = (4 - 1)xy + (-3 - 4)zx + (4 + 3)yz + 7$

=> $A = 3xy + (-7)zx + 7yz + 7$

=> A = $3xy - 7zx + 7yz + 7$

Thus, the required number is $3xy - 7zx + 7yz + 7$

Question 18 – What should be subtracted from $x^2 - xy + y^2 - x + y + 3$ to obtain $-x^2 + 3y^2 - 4xy + 1$?

Solution: Let the required number be 'A'

Then according to given question, we have

$(x^2 - xy + y^2 - x + y + 3) - A = -x^2 + 3y^2 - 4xy + 1$

=> A $= (x^2 - xy + y^2 - x + y + 3) - (-x^2 + 3y^2 - 4xy + 1)$

Now, we will solve by using horizontal method

We will change the signs from '-'to '+' and from '+' to '-'in the second bracket as follows:

=> A $= x^2 - xy + y^2 - x + y + 3 + x^2 - 3y^2 + 4xy - 1$

=> A $= (x^2 + x^2) + (y^2 - 3y^2) + (-xy + 4xy) - x + y + (3 - 1)$

=> $A = (1 + 1)x^2 + (1 - 3)y^2 + (-1 + 4)xy - x + y + 2$

=> $A = 2x^2 + (-2)y^2 + 3xy - x + y + 2$

=> $A = 2x^2 - 2y^2 + 3xy - x + y + 2$

Thus, the required number is $2x^2 - 2y^2 + 3xy - x + y + 2$

Question 19 - How much is $x - 2y + 3z$ greater than $3x + 5y - 7$?

Solution: We will solve the following: $(x - 2y + 3z) - (3x + 5y - 7)$

We will change the signs from '-'to '+' and from '+' to '-'in the second bracket as follows:

$= x - 2y + 3z - 3x - 5y + 7$

$= (x - 3x) + (-2y - 5y) + (3z) + 7$

$= (1 - 3)x + (-2 - 5)y + 3z + 7$

$= -2x + (-7)y + 3z + 7$

$= -2x - 7y + 3z + 7$

Question 20 – How much is $x^2 - 2xy + 3y^2$ less than $2x^2 - 3y^2 + xy$?

Solution: We will solve the following: $(2x^2 - 3y^2 + xy) - (x^2 - 2xy + 3y^2)$

We will change the signs from '-'to '+' and from '+' to '-'in the second bracket as follows:

$2x^2 - 3y^2 + xy - x^2 + 2xy - 3y^2$

$= (2x^2 - x^2) + (-3y^2 - 3y^2) + (xy + 2xy)$

$= (2 - 1)x^2 + (-3 - 3)y^2 + (1 + 2)xy$

$= x^2 + (-6)y^2 + 3xy$

$= x^2 - 6y^2 + 3xy$

Question 21 – How much does $a^2 - 3ab + 2b^2$ exceed $2a^2 - 7ab + 9b^2$?

Solution: We will solve the following: $(a^2 - 3ab + 2b^2) - (2a^2 - 7ab + 9b^2)$

We will change the signs from '-'to '+' and from '+' to '-'in the second bracket as follows:

$a^2 - 3ab + 2b^2 - 2a^2 + 7ab - 9b^2$

$= (a^2 - 2a^2) + (2b^2 - 9b^2) + (-3ab + 7ab)$

$= (1 - 2)a^2 + (2 - 9)b^2 + (-3 + 7)ab$

$= -a^2 + (-7)b^2 + 4ab$

$= -a^2 - 7b^2 + 4ab$

Question 22 – What must be added to $12x^3 - 4x^2 + 3x - 7$ to make the sum $x^3 + 2x^2 - 3x + 2$?

Solution: Let the required number be 'A'

Then according to given question, we have

$(12x^3 - 4x^2 + 3x - 7) + A = x^3 + 2x^2 - 3x + 2$

$\Rightarrow A = (x^3 + 2x^2 - 3x + 2) - (12x^3 - 4x^2 + 3x - 7)$

Now, we will solve by using horizontal method

We will change the signs from '-'to '+' and from '+' to '-'in the second bracket as follows:

$\Rightarrow A = x^3 + 2x^2 - 3x + 2 - 12x^3 + 4x^2 - 3x + 7$

$\Rightarrow A = (x^3 - 12x^3) + (2x^2 + 4x^2) + (-3x - 3x) + (2 + 7)$

$\Rightarrow A = (1 - 12)x^3 + (2 + 4)x^2 + (-3 - 3)x + 9$

$\Rightarrow A = -11x^3 + 6x^2 + (-6)x + 9$

$\Rightarrow A = -11x^3 + 6x^2 - 6x + 9$

Thus, the required number is $-11x^3 + 6x^2 - 6x + 9$

Question 23 – If $P = 7x^2 + 5xy - 9y^2$, $Q = 4y^2 - 3x^2 - 6xy$ and $R = -4x^2 + xy + 5y^2$,

Show that $P + Q + R = 0$.

Solution: We will consider LHS = $P + Q + R$

$\Rightarrow (7x^2 + 5xy - 9y^2) + (4y^2 - 3x^2 - 6xy) + (-4x^2 + xy + 5y^2)$

We collect like terms in separate brackets and solve as follows:

$= (7x^2 - 3x^2 - 4x^2) + (-9y^2 + 4y^2 + 5y^2) + (5xy - 6xy + xy)$

$= (7 - 3 - 4)x^2 + (-9 + 4 + 5)y^2 + (5 - 6 + 1)xy$

$= 0x^2 + 0x + 0xy$

= 0 = RHS

Hence, $P + Q + R = 0$

Question 24 – If $P = a^2 - b^2 + 2ab$, $Q = a^2 + 4b^2 - 6ab$, $R = b^2 + b$, $S = a^2 - 4ab$ and $T = -2a^2 + b^2 - ab + a$. Find $P + Q + R + S - T$.

Solution: We will consider $P + Q + R + S - T$

$((a^2 - b^2 + 2ab) + (a^2 + 4b^2 - 6ab) + (b^2 + b) + (a^2 - 4ab)) - (-2a^2 + b^2 - ab + a)$

We will change the signs from'-'to '+' and from'+' to '-'in the last bracket as follows:

$a^2 - b^2 + 2ab + a^2 + 4b^2 - 6ab + b^2 + b + a^2 - 4ab + 2a^2 - b^2 + ab - a)$

Now, we collect like terms in separate brackets and solve as follows:

$= (a^2 + a^2 + a^2 + 2a^2) + (-b^2 + 4b^2 + b^2 - b^2) + (2ab - 6ab - 4ab + ab) + b - a$

$= (1 + 1 + 1 + 2)a^2 + (-1 + 4 + 1 - 1)b^2 + (2 - 6 - 4 + 1)ab + b - a$

$= 5a^2 + 3b^2 + (-7)ab + b - a$

$= 5a^2 + 3b^2 - 7ab + b - a$

Therefore, $P + Q + R + S - T = 5a^2 + 3b^2 - 7ab + b - a$

Use of brackets in writing algebraic expressions:

It is very important to use brackets or grouping symbols while solving the algebraic expression involving two or more terms. There are different types of brackets used in operations on algebraic expressions which are as follows:

(1) Parentheses ( )
(2) Curly brackets of braces { }
(3) Square brackets [ ]

Examples:

Example 1 – Put the last two terms of each of the following expressions in the parentheses preceded by a minus sign:

(i) $2x + 3y - 4z + 7$

Solution: We know that when we subtract two algebraic expressions, then sign changes in the second bracket. Now, if we want to put bracket preceded by minus sign, we have to follow the same rule.

Thus, we can write: $2x + 3y - 4z + 7$

$= 2x + 3y - (4z - 7)$

(ii) $3a - 2b - 7c - 4d$

Solution: We know that when we subtract two algebraic expressions, then sign changes in the second bracket. Now, if we want to put bracket preceded by minus sign, we have to follow the same rule.

Thus, we can write: $3a - 2b - 7c - 4d$

$= 3a - 2b - (7c + 4d)$

(iii) $7xy - 4yz + 3zx - 5$

Solution: We know that when we subtract two algebraic expressions, then sign changes in the second bracket. Now, if we want to put bracket preceded by minus sign, we have to follow the same rule.

Thus, we can write: $7xy - 4yz + 3zx - 5$

$= 7xy - 4yz - (-3zx + 5)$

Example 2 – Write each of the following statements by using appropriate grouping symbols:

(i) The sum of $x + y$ *and* $2xy - 3x + 2y$ is subtracted from $xy - x + y$

Solution: Sum of $(x + y)$ *and* $(2xy - 3x + 2y)$ is represented as $(x + y) + (2xy - 3x + 2y)$

Now, it is to be subtracted from $(xy - x + y)$

Thus, by using grouping symbols, we have

$(xy - x + y) - \{(x + y) + (2xy - 3x + 2y)\}$

(ii) The subtraction of $x + y - 3$ from $3x - 2y + 9$ is subtracted from the sum of $4x + 3y - 9$ and $2x - y + z$

Solution: Firstly, subtraction of $x + y - 3$ from $3x - 2y + 9$ is represented as:

$(3x - 2y + 9) - (x + y - 3)$

Sum of $4x + 3y - 9$ and $2x - y + z$ is represented as:

$(4x + 3y - 9) + (2x - y + z)$

Now according to the given question we have,

$\{(4x + 3y - 9) + (2x - y + z)\} - \{(3x - 2y + 9) - (x + y - 3)\}$

(iii) The subtraction of $y - 1$ from x is added to $3y$ and its difference from y is subtracted from x.

Solution: Firstly, subtraction of $y - 1$ from x is represented as:

$x - (y - 1)$

Now, it is to be added to $3y$

=> $x - (y - 1) + 3y$

Its difference from y is: $y - \{x - (y - 1) + 3y\}$

Now when it is subtracted from x, we get,

$x - [\, y - \{x - (y - 1) + 3y\}]$

Example 3 – Place the last two terms in each of the following expressions in parentheses preceded by a '-'sign:

(i) $9a + 5xy - 7x^2 + 8y - 6$

Solution: We know that when we subtract two algebraic expressions, then sign changes in the second bracket. Now, if we want to put bracket preceded by minus sign, we have to follow the same rule.

Thus, we can write: $9a + 5xy - 7x^2 + 8y - 6$

$= 9a + 5xy - 7x^2 - (-8y + 6)$

(ii) $-y + z + x^2 - y^2 - a^2$

Solution: We know that when we subtract two algebraic expressions, then sign changes in the second bracket. Now, if we want to put bracket preceded by minus sign, we have to follow the same rule.

Thus, we can write: $-y + z + x^2 - y^2 - a^2$

$= -y + z + x^2 - (y^2 + a^2)$

(iii) $x + \ y + z - xy - yz - zx$

Solution: We know that when we subtract two algebraic expressions, then sign changes in the second bracket. Now, if we want to put bracket preceded by minus sign, we have to follow the same rule.

Thus, we can write: $x + \ y + z - xy - yz - zx$

$= x + \ y + z - xy - (yz + zx) = x + \ y + z - xy - z(y + x)$

(iv) $xy^2 + yz^2 + zx^2$

Solution: We know that when we subtract two algebraic expressions, then sign changes in the second bracket. Now, if we want to put bracket preceded by minus sign, we have to follow the same rule.

Thus, we can write: $xy^2 + yz^2 + zx^2$

= $xy^2 - (-yz^2 - zx^2)$

= $xy^2 - z(-yz - x^2)$

Exercise 7.3

Question 1 – Place the last two terms of the following expressions in parentheses preceded by a minus sign:

(i) $x + y - 3z + y$

Solution: We know that when we subtract two algebraic expressions, then sign changes in the second bracket. Now, if we want to put bracket preceded by minus sign, we have to follow the same rule.

Thus, we can write: $x + y - 3z + y$

= $x + y - (3z - y)$

(ii) $3x - 2y - 5z - 4$

Solution: We know that when we subtract two algebraic expressions, then sign changes in the second bracket. Now, if we want to put bracket preceded by minus sign, we have to follow the same rule.

Thus, we can write: $3x - 2y - 5z - 4$

= $3x - 2y - (5z + 4)$

(iii) $3a - 2b + 4c - 5$

Solution: We know that when we subtract two algebraic expressions, then sign changes in the second bracket. Now, if we want to put bracket preceded by minus sign, we have to follow the same rule.

Thus, we can write: $3a - 2b + 4c - 5$

$= 3a - 2b - (-4c + 5)$

(iv) $7a + 3b + 2c + 4$

Solution: We know that when we subtract two algebraic expressions, then sign changes in the second bracket. Now, if we want to put bracket preceded by minus sign, we have to follow the same rule.

Thus, we can write: $7a + 3b + 2c + 4$

$= 7a + 3b - (-2c - 4)$

(v) $2a^2 - b^2 - 3ab + 6$

Solution: We know that when we subtract two algebraic expressions, then sign changes in the second bracket. Now, if we want to put bracket preceded by minus sign, we have to follow the same rule.

Thus, we can write: $2a^2 - b^2 - 3ab + 6$

$= 2a^2 - b^2 - (3ab - 6)$

(vi) $a^2 + b^2 - c^2 + ab - 3ac$

Solution: We know that when we subtract two algebraic expressions, then sign changes in the second bracket. Now, if we want to put bracket preceded by minus sign, we have to follow the same rule.

Thus, we can write: $a^2 + b^2 - c^2 + ab - 3ac$

$= a^2 + b^2 - c^2 - (-ab + 3ac)$

$= a^2 + b^2 - c^2 - a(-b + 3c)$

Question 2 – Write each of the following statements by using appropriate grouping symbols:

(i) The sum of $a - b$ and $3a - 2b + 5$ is subtracted from $4a + 2b - 7$.

Solution: Sum of $(a - b)\ and\ (3a - 2b + 5)$ is represented as $(a - b) + (3a - 2b + 5)$

Now, it is to be subtracted from $(4a + 2b - 7)$

Thus, by using grouping symbols, we have

$(4a + 2b - 7) - \{(a - b) + (3a - 2b + 5)\}$

(ii) Three times the sum of $2x + y - \{5 - (x - 3y)\}$ and $7x - 4y + 3$ is subtracted from $3x - 4y + 7$

Solution: Sum of $2x + y - \{5 - (x - 3y)\}\ and\ (7x - 4y + 3)$ is represented as:

$2x + y - \{5 - (x - 3y)\} + (7x - 4y + 3)$

Now, three times of this is represented as:

$3[(2x + y - \{5 - (x - 3y)\}) + (7x - 4y + 3)]$

Now, it is to be subtracted from $3x - 4y + 7$

$\Rightarrow (3x - 4y + 7) - 3[(2x + y - \{5 - (x - 3y)\}) + (7x - 4y + 3)]$

(iii) The subtraction of $x^2 - y^2 + 4xy$ from $2x^2 + y^2 - 3xy$ is added to $9x^2 - 3y^2 - xy$.

Solution: Subtraction of $x^2 - y^2 + 4xy$ from $2x^2 + y^2 - 3xy$ is represented as:

$(2x^2 + y^2 - 3xy) - (x^2 - y^2 + 4xy)$

Now, it is added to $9x^2 - 3y^2 - xy$

$\Rightarrow \{(2x^2 + y^2 - 3xy) - (x^2 - y^2 + 4xy)\} + (9x^2 - 3y^2 - xy)$

Removal of brackets:

There are some rules while solving the operations on algebraic expressions:

1) When there is a '+' sign preceding a bracket or grouping symbol, then we remove the bracket and there is no change in the sign of terms.

2) When there is a '-'sign preceding a bracket or grouping symbol, then we remove the bracket and sign will change from '+' to '-' and from '-' to '+'.

Examples:

Example 1 – Simplify each of the following algebraic expressions:

(i) $(\boldsymbol{a^2 + b^2 + 2ab}) + (\boldsymbol{a^2 + b^2 - 2ab})$

Solution: Here, we can see that there is '+' sign preceding the bracket thus there is no change in sign.

=> $(a^2 + b^2 + 2ab) + (a^2 + b^2 - 2ab)$

= $a^2 + b^2 + 2ab + a^2 + b^2 - 2ab$

Now, we collect like terms and solve as follows:

= $(a^2 + a^2) + (b^2 + b^2) + (2ab - 2ab)$

= $(1 + 1)a^2 + (1 + 1)b^2 + (2 - 2)ab$

= $2a^2 + 2b^2 + 0ab$

= $2a^2 + 2b^2$

(ii) $(\boldsymbol{a^2 + b^2 + 2ab}) - (\boldsymbol{a^2 + b^2 - 2ab})$

Solution: Here, we can see that there is '-' sign preceding the bracket thus signs will get change in the second bracket.

=> $(a^2 + b^2 + 2ab) - (a^2 + b^2 - 2ab)$

$= a^2 + b^2 + 2ab - a^2 - b^2 + 2ab$

Now, we collect like terms and solve as follows:

$= (a^2 - a^2) + (b^2 - b^2) + (2ab + 2ab)$

$= (1 - 1)a^2 + (1 - 1)b^2 + (2 + 2)ab$

$= 0a^2 + 0b^2 + 4ab$

$= 4ab$

Example 2 – Simplify each of the following:

(i) $\mathbf{-5(a+b) + 2(2a-b) + 4a - 7}$

Solution: Here, we can see that there are both '-' and '+' signs preceding the bracket thus signs will get change in case of '-'and remains unchanged in case of '+'

$=> -5(a+b) + 2(2a-b) + 4a - 7$

$= -5a - 5b + 4a - 2b + 4a - 7$

Now, we collect like terms and solve as follows:

$= (-5a + 4a + 4a) + (-5b - 2b) + (-7)$

$= (-5 + 4 + 4)a + (-5 - 2)b + (-7)$

$= 3a + (-7)b + (-7)$

$= 3a - 7b - 7$

(ii) $\mathbf{-3(a+b) + 4(2a-3b) - (2a-b)}$

Solution: Here, we can see that there are both '-' and '+' signs preceding the bracket thus signs will get change in case of '-'and remains unchanged in case of '+'

$=> -3(a+b) + 4(2a-3b) - (2a-b)$

$= -3a - 3b + 8a - 12b - 2a + b$

Now, we collect like terms and solve as follows:

$= (-3a + 8a - 2a) + (-3b - 12b + b)$

$= (-3 + 8 - 2)a + (-3 - 12 + 1)b$

$= 3a + (-14)b$

$= 3a - 14b$

Example 3 – Simplify each of the following:

(i) $\boldsymbol{2x - \{5y - (x - 2y)\}}$

Solution: Here, we can see that there are more than one grouping symbols thus we firstly remove the innermost bracket and solve accordingly.

$=> 2x - \{5y - (x - 2y)\}$

$= 2x - \{5y - x + 2y\}$

$= 2x - \{5y + 2y - x\}$

$= 2x - \{7y - x\}$

$= 2x - 7y + x$

$= 2x + x - 7y$

$= 3x - 7y$

(ii) $\boldsymbol{2x - [3y - \{2x - (y - x)\}]}$

Solution: Here, we can see that there are more than one grouping symbols thus we firstly remove the innermost bracket and solve accordingly.

$=> 2x - [3y - \{2x - (y - x)\}]$

$= 2x - [3y - \{2x - y + x\}]$

$= 2x - [3y - \{2x + x - y\}]$

$= 2x - [3y - \{3x - y\}]$

$= 2x - [3y - 3x + y]$

$= 2x - [3y + y - 3x]$

$= 2x - [4y - 3x]$

$= 2x - 4y + 3x$

$= 2x + 3x - 4y$

$= 5x - 4y$

(iii) $-\boldsymbol{m} - [\boldsymbol{m} + \{\boldsymbol{m} + \boldsymbol{n} - \boldsymbol{2m} - (\boldsymbol{m} - \boldsymbol{2n})\} - \boldsymbol{n}]$

Solution: Here, we can see that there are more than one grouping symbols thus we firstly remove the innermost bracket and solve accordingly.

$=> -m - [m + \{m + n - 2m - (m - 2n)\} - n]$

$= -m - [m + \{m + n - 2m - m + 2n\} - n]$

$= -m - [m + \{m - 2m - m + n + 2n\} - n]$

$= -m - [m + \{-2m + 3n\} - n]$

$= -m - [m - 2m + 3n - n]$

$= -m - [-m + 2n]$

$= -m + m - 2n$

$= -2n$

(iv) $3x^2z - 4yz + 3xy - \{x^2z - (x^2z - 3yz) - 4yz - 7z\}$

Solution: Here, we can see that there are more than one grouping symbols thus we firstly remove the innermost bracket and solve accordingly.

=> $3x^2z - 4yz + 3xy - \{x^2z - (x^2z - 3yz) - 4yz - 7z\}$

= $3x^2z - 4yz + 3xy - \{x^2z - x^2z + 3yz - 4yz - 7z\}$

= $3x^2z - 4yz + 3xy - \{-yz - 7z\}$

= $3x^2z - 4yz + 3xy + yz + 7z$

= $3x^2z - 4yz + yz + 3xy + 7z$

= $3x^2z - 3yz + 3xy + 7z$

Example 4 – Simplify: $15x - [8x^3 + 3x^2 - \{8x^2 - (4 - 2x - x^3) - 5x^3\} - 2x]$

Solution: Here, we can see that there are more than one grouping symbol thus we firstly remove the innermost bracket and solve accordingly.

=> $15x - [8x^3 + 3x^2 - \{8x^2 - (4 - 2x - x^3) - 5x^3\} - 2x]$

= $15x - [8x^3 + 3x^2 - \{8x^2 - 4 + 2x + x^3 - 5x^3\} - 2x]$

= $15x - [8x^3 + 3x^2 - \{8x^2 - 4 + 2x - 4x^3\} - 2x]$

= $15x - [8x^3 + 3x^2 - 8x^2 + 4 - 2x + 4x^3 - 2x]$

= $15x - [8x^3 + 4x^3 + 3x^2 - 8x^2 + 4 - 2x - 2x]$

= $15x - [12x^3 - 5x^2 + 4 - 4x\ ]$

= $15x - 12x^3 + 5x^2 - 4 + 4x$

= $-\ 12x^3 + 5x^2 + 15x + 4x - 4$

= $-\ 12x^3 + 5x^2 + 19x - 4$

Example 5 – Simplify: $\mathbf{5 + [x - \{2y - (6x + y - 4) + 2x^2\} - (x^2 - 2y)]}$

Solution: Here, we can see that there are more than one grouping symbol thus we firstly remove the innermost bracket and solve accordingly.

=> $5 + [x - \{2y - (6x + y - 4) + 2x^2\} - (x^2 - 2y)]$

= $5 + [x - \{2y - 6x - y + 4 + 2x^2\} - (x^2 - 2y)]$

= $5 + [x - \{2y - y - 6x + 4 + 2x^2\} - (x^2 - 2y)]$

= $5 + [x - \{y - 6x + 4 + 2x^2\} - (x^2 - 2y)]$

= $5 + [x - y + 6x - 4 - 2x^2 - x^2 + 2y]$

= $5 + [x + 6x - y + 2y - 4 - 2x^2 - x^2]$

= $5 + [7x + y - 4 - 3x^2]$

= $5 + 7x + y - 4 - 3x^2$

= $5 - 4 + 7x + y - 3x^2$

= $1 + 7x + y - 3x^2$

Example 6 – Simplify and find the value of the following expression when $\mathbf{a = 3 \; and \; b = 1}$

$\mathbf{4(a^2 + b^2 + 2ab) - [4(a^2 + b^2 - 2ab) - \{-b^3 + 4(a - 3)\}]}$

Solution: We will first simplify the given expression as follows:

Here, we can see that there are more than one grouping symbol thus we firstly remove the innermost bracket and solve accordingly.

=> $4(a^2 + b^2 + 2ab) - [4(a^2 + b^2 - 2ab) - \{-b^3 + 4(a - 3)\}]$

= $4a^2 + 4b^2 + 8ab - [4(a^2 + b^2 - 2ab) - \{-b^3 + 4a - 12\}]$

= $4a^2 + 4b^2 + 8ab - [4(a^2 + b^2 - 2ab) + b^3 - 4a + 12]$

$= 4a^2 + 4b^2 + 8ab - [4a^2 + 4b^2 - 8ab + b^3 - 4a + 12]$

$= 4a^2 + 4b^2 + 8ab - 4a^2 - 4b^2 + 8ab - b^3 + 4a - 12$

$= 4a^2 - 4a^2 + 4b^2 - 4b^2 + 8ab + 8ab - b^3 + 4a - 12$

$= 16ab - b^3 + 4a - 12$

Now, we will put a = 3 and b = 1 in the above expression

$= 16(3)(1) - (1)^3 + 4(3) - 12$

$= 48 - 1 + 12 - 12$

$= 47$

Exercise 7.4

Simplify each of the following algebraic expressions by removing grouping symbols:

Question 1: $2x + (5x - 3y)$

Solution: Here, we can see that there is '+' sign preceding the bracket thus there is no change in sign.

$=> 2x + (5x - 3y)$

$= 2x + 5x - 3y$

$= 7x - 3y$

Question 2: $3x - (y - 2x)$

Solution: Here, we can see that there is '-' sign preceding the bracket thus signs will get change in the second bracket.

$=> 3x - (y - 2x)$

$= 3x - y + 2x$

$= 3x + 2x - y$

$= 5x - y$

Question 3: $5a - (3b - 2a + 4c)$

Solution: Here, we can see that there is '-' sign preceding the bracket thus signs will get change in the second bracket.

$=> 5a - (3b - 2a + 4c)$

$= 5a - 3b + 2a - 4c$

$= 5a + 2a - 3b - 4c$

$= 7a - 3b - 4c$

Question 4: $-2(x^2 - y^2 + xy) - 3(x^2 + y^2 - xy)$

Solution: Here, we can see that there is '-' sign preceding the bracket thus signs will get change in the second bracket.

$=> -2(x^2 - y^2 + xy) - 3(x^2 + y^2 - xy)$

$= -2x^2 + 2y^2 - 2xy - 3x^2 - 3y^2 + 3xy$

$= -2x^2 - 3x^2 + 2y^2 - 3y^2 - 2xy + 3xy$

$= -5x^2 - y^2 + xy$

Question 5: $3x + 2y - \{x - (2y - 3)\}$

Solution: Here, we can see that there are more than one grouping symbols thus we firstly remove the innermost bracket and solve accordingly.

$=> 3x + 2y - \{x - (2y - 3)\}$

$= 3x + 2y - \{x - 2y + 3\}$

$= 3x + 2y - x + 2y - 3$

$= 3x - x + 2y + 2y - 3$

$= 2x + 4y - 3$

Question 6: $5a - \{3a - (2 - a) + 4\}$

Solution: Here, we can see that there are more than one grouping symbols thus we firstly remove the innermost bracket and solve accordingly.

$=> 5a - \{3a - (2 - a) + 4\}$

$= 5a - \{3a - 2 + a + 4\}$

$= 5a - \{3a + a - 2 + 4\}$

$= 5a - \{4a + 2\}$

$= 5a - 4a - 2$

$= a - 2$

Question 7: $a - [b - \{a - (b - 1) + 3a\}]$

Solution: Here, we can see that there are more than one grouping symbols thus we firstly remove the innermost bracket and solve accordingly.

$=> a - [b - \{a - (b - 1) + 3a\}]$

$= a - [b - \{a - b + 1 + 3a\}]$

$= a - [b - \{a + 3a - b + 1\}]$

$= a - [b - \{4a - b + 1\}]$

$= a - [b - 4a + b - 1]$

$= a - [b + b - 4a - 1]$

$= a - [2b - 4a - 1]$

$= a - 2b + 4a + 1$

$= a + 4a - 2b + 1$

$= 5a - 2b + 1$

Question 8: $\boldsymbol{a - [2b - \{3a - (2b - 3c)\}]}$

Solution: Here, we can see that there are more than one grouping symbols thus we firstly remove the innermost bracket and solve accordingly.

$=> a - [2b - \{3a - (2b - 3c)\}]$

$= a - [2b - \{3a - 2b + 3c\}]$

$= a - [2b - 3a + 2b - 3c]$

$= a - [2b + 2b - 3a - 3c]$

$= a - [4b - 3a - 3c]$

$= a - 4b + 3a + 3c$

$= a + 3a - 4b + 3c$

$= 4a - 4b + 3c$

Question 9: $\boldsymbol{-x + [5y - \{2x - (3y - 5x)\}]}$

Solution: Here, we can see that there are more than one grouping symbols thus we firstly remove the innermost bracket and solve accordingly.

$=> -x + [5y - \{2x - (3y - 5x)\}]$

$= -x + [5y - \{2x - 3y + 5x\}]$

$= -x + [5y - \{2x + 5x - 3y\}]$

$= -x + [5y - \{7x - 3y\}]$

$= -x + [5y - 7x + 3y]$

$= -x + [5y + 3y - 7x]$

$= -x + [8y - 7x]$

$= -x + 8y - 7x$

$= -x - 7x + 8y$

$= -8x + 8y$

Question 10: $2a - [4b - \{4a - 3(2a - b)\}]$

Solution: Here, we can see that there are more than one grouping symbols thus we firstly remove the innermost bracket and solve accordingly.

$=> 2a - [4b - \{4a - 3(2a - b)\}]$

$= 2a - [4b - \{4a - 6a + 3b\}]$

$= 2a - [4b - \{-2a + 3b\}]$

$= 2a - [4b + 2a - 3b]$

$= 2a - [4b - 3b + 2a]$

$= 2a - [b + 2a]$

$= 2a - b - 2a$

$= 2a - 2a - b$

$= -b$

Question 11: $-a - [a + \{a + b - 2a - (a - 2b)\} - b]$

Solution: Here, we can see that there are more than one grouping symbols thus we firstly remove the innermost bracket and solve accordingly.

$=> -a - [a + \{a + b - 2a - (a - 2b)\} - b]$

$= -a - [a + \{a + b - 2a - a + 2b\} - b]$

$= -a - [a + \{a - a - 2a + b + 2b\} - b]$

$= -a - [a + \{-2a + 3b\} - b]$

$= -a - [a - 2a + 3b - b]$

$= -a - [-a + 2b]$

$= -a + a - 2b$

$= -2b$

Question 12: $2x - 3y - [3x - 2y - \{x - z - (x - 2y)\}]$

Solution: Here, we can see that there are more than one grouping symbols thus we firstly remove the innermost bracket and solve accordingly.