Introduction

Cube of a number: for a given number x, we define cube of x = $x \times x \times x$ and is denoted byx^3.

Perfect cube: A natural number is said to be a perfect cube if it is the cube of some natural number.

Note: A given number is a perfect cube if it can be expressed as the product of triplets of equal factors.

Cube of a rational number:

$$\left(\frac{a}{b}\right)^3 = \frac{a^3}{b^3}$$

Properties of cubes of numbers

(a) The cube of every even number is even.

(b) The cube of every odd number is odd.

Examples:

Example 1 - Show that 189 is not a perfect cube.

Solution **-** First we resolve 189 into prime factors:

3	189
3	63
3	21
7	7
	1

We can see 189 = $\underline{3 \times 3 \times 3} \times 7$

Here one triplet is formed and we are left with one more factor

Thus, 189 cannot be expressed as a product of triplets

Hence, 189 is not a perfect cube.

Example 2 - Show that 216 is a perfect cube. Find the number whose cube is 216.

Solution - First we resolve 216 into prime factors:

2	216
2	108
2	54
3	27
3	9
3	3
	1

We can see 216 = $\underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}$

$$= 2^3 \times 3^3 = (2 \times 3)^3 = 6^3$$

Since 216 can be expressed as a product of triplets

Hence, 216 is a perfect cube.

And 6 is the number whose cube is 216.

Example 3 - What is the smallest number by which 3087 may be multiplied so that the product is a perfect cube?

Solution - First we resolve 3087 into prime factors:

3	3087
3	1029
7	343
7	49
7	7
	1

We have, 3087 = $\underline{7 \times 7 \times 7} \times 3 \times 3$

Since it can be seen that 3087 must be multiplied by 3 to be a perfect cube

Example 4 - What is the smallest number by which 392 may be divided so that the quotient is a perfect cube?

Solution - First we resolve 392 into prime factors:

2	392
2	196
2	98
7	49
7	7
	1

We have, 392 = $2 \times 2 \times 2 \times 7 \times 7$

Since it can be seen that 392 must be divided by (7×7) to be a perfect cube

=> 49 is the smallest number by which 392 must be divided so that quotient is a perfect cube

Example 5 - Find the cube of each of the following:

(a) (-7)

Solution: $(-7)^3 = (-7) \times (-7) \times (-7) = -343$

(b) $1\frac{2}{3}$

Solution: $\left(1\frac{2}{3}\right)^3 = \left(\frac{5}{3}\right)^3 = \frac{5^3}{3^3} = \frac{5\times5\times5}{3\times3\times3} = \frac{125}{27}$

(c) 2.5

Solution: $(2.5)^3 = (\frac{25}{10})^3 = (\frac{5}{2})^3 = \frac{5^3}{2^3} = \frac{5\times5\times5}{2\times2\times2} = \frac{125}{8}$

(d) 0.06

Solution: $(0.06)^3 = (\frac{6}{100})^3 = (\frac{3}{50})^3 = \frac{3^3}{(50)^3} = \frac{3\times3\times3}{50\times50\times50} = \frac{27}{125000}$

Exercise 4A

Question 1 - Evaluate:

(a) $(8)^3$

Solution: $8^3 = 8 \times 8 \times 8 = 512$

(b) $(15)^3$

Solution: $(15)^3 = 15 \times 15 \times 15 = 3375$

(c) $\mathbf{(21)^3}$

Solution: $(21)^3 = 21 \times 21 \times 21 = 9261$

(d) $\mathbf{(60)^3}$

Solution: $(60)^3 = 60 \times 60 \times 60 = 216000$

Question 2 - Evaluate:

(a) $\mathbf{(1.2)^3}$

Solution: $(1.2)^3 = (\frac{12}{10})^3 = (\frac{6}{5})^3 = \frac{6^3}{5^3} = \frac{6\times6\times6}{5\times5\times5} = \frac{216}{125}$

(b) $\mathbf{(3.5)^3}$

Solution: $(3.5)^3 = (\frac{35}{10})^3 = (\frac{7}{2})^3 = \frac{7^3}{2^3} = \frac{7\times7\times7}{2\times2\times2} = \frac{343}{8}$

(c) $\mathbf{(0.8)^3}$

Solution: $(0.8)^3 = (\frac{8}{10})^3 = (\frac{4}{5})^3 = \frac{4^3}{5^3} = \frac{4\times4\times4}{5\times5\times5} = \frac{64}{125}$

(d) $\mathbf{(0.05)^3}$

Solution: $(0.05)^3 = (\frac{5}{100})^3 = (\frac{1}{20})^3 = \frac{1^3}{(20)^3} = \frac{1\times1\times1}{20\times20\times20} = \frac{1}{8000}$

(e) $\mathbf{(1.2)^3}$

Solution: $(1.2)^3 = (\frac{12}{10})^3 = (\frac{6}{5})^3 = \frac{6^3}{5^3} = \frac{6\times6\times6}{5\times5\times5} = \frac{216}{125}$

Question 3 - Evaluate:

(a) $\mathbf{(\frac{4}{7})^3}$

Solution: $\left(\frac{4}{7}\right)^3 = \frac{4^3}{7^3} = \frac{4\times4\times4}{7\times7\times7} = \frac{64}{343}$

(b) $\mathbf{(\frac{10}{11})^3}$

Solution: $\left(\frac{10}{11}\right)^3 = \frac{10^3}{11^3} = \frac{10\times10\times10}{11\times11\times11} = \frac{1000}{1331}$

(c) $(\frac{1}{15})^3$

Solution: $\left(\frac{1}{15}\right)^3 = \frac{1^3}{(15)^3} = \frac{1\times1\times1}{15\times15\times15} = \frac{1}{3375}$

(d) $(1\frac{3}{10})^3$

Solution: $\left(1\frac{3}{10}\right)^3 = \left(\frac{13}{10}\right)^3 = \frac{(13)^3}{(10)^3} = \frac{13\times13\times13}{10\times10\times10} = \frac{2197}{1000}$

Question 4 – Which of the following numbers are perfect cubes? In case of perfect cube, find the number whose cube is the given number.

(a) 125

Solution - First we resolve 125 into prime factors:

5	125
5	25
5	5
	1

We can see $125 = 5 \times 5 \times 5$

$= 5^3$

Since 125 can be expressed as a product of triplet

Hence, 125 is a perfect cube.

And 5 is the number whose cube is 125.

(b) 243

Solution - First we resolve 243 into prime factors:

3	243
3	81
3	27
3	9
3	3
	1

We can see 243 = $3 \times 3 \times 3 \times 3 \times 3$

Since 243 cannot be expressed as a product of triplets

Hence, 243 is not a perfect cube.

(c) 343

Solution - First we resolve 343 into prime factors:

7	343
7	49
7	7
	1

We can see 343 = $7 \times 7 \times 7$

$= 7^3$

Since 343 can be expressed as a product of triplet

Hence, 343 is a perfect cube.

And 7 is the number whose cube is 343.

(d) 256

Solution - First we resolve 256 into prime factors:

2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

We can see 256 = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Since 256 cannot be expressed as a product of triplets

Hence, 256 is not a perfect cube.

(e) 8000

Solution - First we resolve 8000 into prime factors:

2	8000
2	4000
2	2000
2	1000
2	500
2	250
5	125
5	25
5	5
	1

We can see 8000 = $\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5}$

$$= 2^3 \times 2^3 \times 5^3 = (2 \times 2 \times 5)^3 = (20)^3$$

Since 8000 can be expressed as a product of triplets

Hence, 8000 is a perfect cube.

And 20 is the number whose cube is 8000.

(f) 9261

Solution - First we resolve 9261 into prime factors:

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

We can see 9261 = $\underline{3 \times 3 \times 3} \times \underline{7 \times 7 \times 7}$

$$= 3^3 \times 7^3 = (3 \times 7)^3 = (21)^3$$

Since 9261 can be expressed as a product of triplets

Hence, 9261 is a perfect cube.

And 21 is the number whose cube is 9261.

(g) 5324

Solution - First we resolve 5324 into prime factors:

2	5324
2	2662
11	1331
11	121
11	11
	1

We can see 5324 = $\underline{11 \times 11 \times 11} \times 2 \times 2$

Since 5324 cannot be expressed as a product of triplets

Hence, 5324 is not a perfect cube.

(h) 3375

Solution - First we resolve 3375 into prime factors:

3	3375
3	1125
3	375
5	125
5	25
5	5
	1

We can see 3375 = $\underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}$

$= 3^3 \times 5^3 = (3 \times 5)^3 = (15)^3$

Since 3375 can be expressed as a product of triplets

Hence, 3375 is a perfect cube.

And 15 is the number whose cube is 3375.

Question 5 - Which of the following are the cubes of even numbers?

(a) 216
(b) 729
(c) 512
(d) 3375

(e) 1000

Solution - Since the cube of every even number is even.

Thus, 216, 512 and 1000 are the cubes of even numbers.

Question 6 - Which of the following are the cubes of odd numbers?

(a) 125
(b) 343
(c) 1728
(d) 4096
(e) 9261

Solution - Since the cube of every odd number is odd.

Thus, 125, 343 and 9261 are the cubes of even numbers.

Question 7 - Find the smallest number by which 1323 must be multiplied so that the product is a perfect cube.

Solution - First we resolve 1323 into prime factors:

3	1323
3	441
3	147
7	49
7	7
	1

We have, 1323 = $\underline{3 \times 3 \times 3} \times 7 \times 7$

Since it can be seen that 1323 must be multiplied by 7 to be a perfect cube

Question 8 - Find the smallest number by which 2560 must be multiplied so that the product is a perfect cube

Solution - First we resolve 2560 into prime factors:

2	2560
2	1280
2	640
2	320
2	160
2	80
2	40
2	20
2	10
5	5
	1

We have, 2560 = $\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 5$

Since it can be seen that 2560 must be multiplied by 5 to be a perfect cube

Question 9 - What is the smallest number by which 1600 must be divided so that the quotient is a perfect cube?

Solution - First we resolve 1600 into prime factors:

2	1600
2	800
2	400
2	200
2	100
2	50
5	25
5	5
	1

We have, 1600 = $\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times 5 \times 5$

Since it can be seen that 1600 must be divided by (5×5) to be a perfect cube

=> 25 is the smallest number by which 1600 must be divided so that quotient is a perfect cube

Question 10 - Find the smallest number by which 8788 must be divided so that the quotient is a perfect cube.

Solution - First we resolve 8788 into prime factors:

2	8788
2	4394
13	2197
13	169
13	13
	1

We have, 8788 = $2 \times 2 \times \underline{13 \times 13 \times 13}$

Since it can be seen that 8788 must be divided by (2×2) to be a perfect cube

=> 4 is the smallest number by which 8788 must be divided so that quotient is a perfect cube.

Short-cut method for finding the cube of a two-digit number

We know that $(a + b)^3 = a^3 + 3a^2b + 3ab^2 + b^3$

Method: For finding the cube of a two digit number with tens digit = a and unit digit=b, we make four columns headed by a^3, $(3a^2$b), $(3ab^2)$, and b^3.

This method is as same as the column method for squaring a two digit number.

a^2 $\times a$ a^3	a^2 $\times 3b$ $3a^2b$	b^2 $\times 3a$ $3b^2a$	b^2 $\times b$ b^3

Example 1 - Find the value of $(29)^3$ by the short cut method.

Solution - We have a = 2 and b = 9

a^2 $\times a$ a^3	a^2 $\times 3b$ $3a^2b$	b^2 $\times 3a$ $3b^2a$	b^2 $\times b$ b^3

4 $\times 2$ 8 +16 24	4 $\times 27$ 108 +55 163	81 $\times 6$ 486 +72 558	81 $\times 9$ 729

Thus, $(29)^3 = 24389$

Example 2 - Find the value of $(71)^3$ by the short cut method.

Solution - We have a = 7 and b = 1

a^2 $\times a$ a^3	a^2 $\times 3b$ $3a^2b$	b^2 $\times 3a$ $3b^2a$	b^2 $\times b$ b^3

49 $\times 7$ 343 +14 357	49 $\times 3$ 147 +2 149	1 $\times 21$ 21	1 $\times 1$ 1

Thus, $(71)^3 = 357911$

Exercise 4B

Find the value of each of the following using the short-cut method:

Question 1 - $(\mathbf{25})^3$

Solution - We have a = 2 and b = 5

a^2 $\times a$ a^3	a^2 $\times 3b$ $3a^2b$	b^2 $\times 3a$ $3b^2a$	b^2 $\times b$ b^3

4 $\times$ 2 8 +7 1<u>5</u>	4 $\times$ 15 60 +16 7<u>6</u>	25 $\times$ 6 150 +12 16<u>2</u>	25 $\times$ 5 12<u>5</u>

Thus, $(25)^3 = 15625$

Question 2 - $(\mathbf{47})^3$

Solution - We have a = 4 and b = 7

a^2 $\times a$ a^3	a^2 $\times 3b$ $3a^2b$	b^2 $\times 3a$ $3b^2a$	b^2 $\times b$ b^3

16 $\times$ 4 64 +39 103	16 $\times$ 21 336 +62 398	49 $\times$ 12 588 +34 622	49 $\times$ 7 343

Thus, $(47)^3 = 103823$

Question 3 - $(68)^3$

Solution - We have a = 6 and b = 8

a^2 $\times a$ a^3	a^2 $\times 3b$ $3a^2b$	b^2 $\times 3a$ $3b^2a$	b^2 $\times b$ b^3

36 $\times$ 6 216 +98 314	36 $\times$ 24 864 +120 984	64 $\times$ 18 1152 +51 1203	64 $\times$ 8 512

Thus, $(68)^3 = 314432$

Question 4 - $(84)^3$

Solution - We have a = 8 and b = 4

a^2 $\times a$ a^3	a^2 $\times 3b$ $3a^2b$	b^2 $\times 3a$ $3b^2a$	b^2 $\times b$ b^3

64 $\times$ 8 512 +80 $\underline{592}$	64 $\times$ 12 768 +39 80$\underline{7}$	16 $\times$ 24 384 +6 39$\underline{0}$	16 $\times$ 4 6$\underline{4}$

Thus, $(84)^3 = 592704$

Cube Roots

The cube root of a number x is that number whose cube gives x. It is denoted by $\sqrt[3]{x}$

Method of finding the cube root of a given number by factorization:

Step1: Express the given number as the product of primes.

Step2: Make groups in triplets of the same prime.

Step3: Find the product of primes, choosing one from each triplet.

Step4: This product is the required cube root of the given number.

Cube root of a negative perfect cube:

$\sqrt[3]{(-a)} = -a$, for any positive integer ‘a’

Cube root of product of integers

$\sqrt[3]{ab} = (\sqrt[3]{a} \times \sqrt[3]{b})$

Cube root of a rational number

$$\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$$

Examples:

Example 1 - Evaluate $\sqrt[3]{\mathbf{216}}$

Solution **-** First we resolve 216 into prime factors:

2	216
2	108
2	54
3	27
3	9
3	3
	1

We see that 216 = 2 × 2 × 2 × 3 × 3 × 3

Thus, $\sqrt[3]{216} = \sqrt[3]{\underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}}$

$= 2 \times 3 = 6$

Example 2 - Evaluate $\sqrt[3]{\mathbf{2744}}$

Solution **-** First we resolve 2744 into prime factors:

2	2744
2	1372
2	686
7	343
7	49
7	7
	1

We see that 2744 = 2 × 2 × 2 × 7 × 7 × 7

Thus, $\sqrt[3]{2744} = \sqrt[3]{\underline{2 \times 2 \times 2} \times \underline{7 \times 7 \times 7}}$

$= 2 \times 7 = 14$

Example 3 - Find the cube root of (-1000)

Solution - We have to find $\sqrt[3]{-1000}$’

Since $\sqrt[3]{-1000} = -\sqrt[3]{1000}$

We resolve 1000 into prime factors:

2	1000
2	500
2	250
5	125
5	25
5	5
	1

We see that 1000 = $2 \times 2 \times 2 \times 5 \times 5 \times 5$

Thus, $\sqrt[3]{1000} = \sqrt[3]{\underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5}}$

= $2 \times 5 = 10$

Thus, $\sqrt[3]{-1000} = -\sqrt[3]{1000} = -10$

Example 4 - Evaluate $\sqrt[3]{\mathbf{125} \times \mathbf{64}}$

Solution - Since $\sqrt[3]{ab} = (\sqrt[3]{a} \times \sqrt[3]{b})$

$\sqrt[3]{125 \times 64} = \sqrt[3]{8000}$

We resolve 8000 into prime factors:

2	8000
2	4000
2	2000
2	1000
2	500
2	250
5	125
5	25
5	5
	1

We can see 8000 = $\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5}$

$= 2^3 \times 2^3 \times 5^3 = (2 \times 2 \times 5)^3 = (20)^3$

Thus, $\sqrt[3]{8000} = \sqrt[3]{20 \times 20 \times 20} = 20$

Example 5 - Evaluate $\sqrt[3]{\mathbf{216} \times (-\mathbf{343})}$

Solution - Since $\sqrt[3]{ab} = (\sqrt[3]{a} \times \sqrt[3]{b})$

Thus, $\sqrt[3]{216 \times (-343)} = \sqrt[3]{216} \times \sqrt[3]{-343}$

We resolve 216 and 343 into prime factors

6	216
6	36
6	6
	1

7	343
7	49
7	7
	1

We see that 216 = 6 × 6 × 6

343 = 7 × 7 × 7

Thus, $\sqrt[3]{216 \times (-343)} = \sqrt[3]{216} \times \sqrt[3]{-343} = \left(\sqrt[3]{6 \times 6 \times 6}\right) \times ((-1)\sqrt[3]{7 \times 7 \times 7})$

$= 6 \times (-7) = -42$

Example 6 - Evaluate

(a) $\sqrt[3]{\frac{\mathbf{216}}{\mathbf{2197}}}$

Solution - Since $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$

Thus, $\sqrt[3]{\frac{216}{2197}} = \frac{\sqrt[3]{216}}{\sqrt[3]{2197}}$

We resolve 216 and 2197 into prime factors

6	216
6	36
6	6
	1

13	2197
13	169
13	13
	1

We see that 216 = 6 × 6 × 6

2197 = 13 × 13 × 13

Thus, $\sqrt[3]{\frac{216}{2197}} = \frac{\sqrt[3]{216}}{\sqrt[3]{2197}} = \frac{\sqrt[3]{6\times6\times6}}{\sqrt[3]{13\times13\times13}} = \frac{6}{13}$

(b) $\sqrt[3]{\frac{-125}{512}}$

Solution - Since $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$

Thus, $\sqrt[3]{\frac{-125}{512}} = \frac{-\sqrt[3]{125}}{\sqrt[3]{512}}$

We resolve 125 and 512 into prime factors

5	125
5	25
5	5
	1

8	512
8	64
8	8
	1

We see that 125 = 5 × 5 × 5

512 = 8 × 8 × 8

Thus, $\sqrt[3]{\frac{-125}{512}} = \frac{-\sqrt[3]{125}}{\sqrt[3]{512}} = \frac{-\sqrt[3]{5\times5\times5}}{\sqrt[3]{8\times8\times8}} = \frac{-5}{8}$

Exercise 4C

Evaluate:

Question 1: $\sqrt[3]{\mathbf{64}}$

Solution - First we resolve 64 into prime factors:

2	64
2	32
2	16
2	8
2	4
2	2
	1

We see that 64 = 2 × 2 × 2 × 2 × 2 × 2

Thus, $\sqrt[3]{64} = \sqrt[3]{\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}}$ = 2 × 2 = 4

Question 2: $\sqrt[3]{\mathbf{343}}$

Solution - First we resolve 343 into prime factors:

7	343
7	49
7	7
	1

We see that 343 = 7 × 7 × 7

Thus, $\sqrt[3]{343} = \sqrt[3]{\underline{7 \times 7 \times 7}}$ = 7

Question 3: $\sqrt[3]{\mathbf{729}}$

Solution - First we resolve 729 into prime factors:

3	729
3	243
3	81
3	27
3	9
3	3
	1

We see that 729 = 3 × 3 × 3 × 3 × 3 × 3

Thus, $\sqrt[3]{729} = \sqrt[3]{\underline{3 \times 3 \times 3} \times \underline{3 \times 3 \times 3}} = 3 \times 3 = 9$

Question 4: $\sqrt[3]{\mathbf{1728}}$

Solution - First we resolve 1728 into prime factors:

2	1728
2	864
2	432
2	216
2	108
2	54
3	27
3	9
3	3
	1

We see that 1728 = 2 × 2 × 2 × 2 × 2 × 2 × 3 × 3 × 3

Thus, $\sqrt[3]{1728} = \sqrt[3]{\underline{3 \times 3 \times 3} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}} = 2 \times 2 \times 3 = 12$

Question 5: $\sqrt[3]{\mathbf{9261}}$

Solution - First we resolve 9261 into prime factors:

3	9261
3	3087
3	1029
7	343
7	49
7	7
	1

We see that 9261 = 7 × 7 × 7 × 3 × 3 × 3

Thus, $\sqrt[3]{9261} = \sqrt[3]{\underline{3 \times 3 \times 3} \times \underline{7 \times 7 \times 7}} = 7 \times 3 = 21$

Question 6: $\sqrt[3]{\mathbf{4096}}$

Solution - First we resolve 4096 into prime factors:

2	4096
2	2048
2	1024
2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

We see that 4096 = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Thus, $\sqrt[3]{4096} = \sqrt[3]{\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}} = 2 \times 2 \times 2 \times 2 = 16$

Question 7: $\sqrt[3]{8000}$

Solution - First we resolve 8000 into prime factors:

2	8000
2	4000
2	2000
2	1000
2	500
2	250
5	125
5	25
5	5
	1

We see that 8000 = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 5 \times 5 \times 5$

Thus, $\sqrt[3]{8000} = \sqrt[3]{\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5}} = 2 \times 2 \times 5 = 20$

Question 8: $\sqrt[3]{3375}$

Solution - First we resolve 3375 into prime factors:

3	3375
3	1125
3	375
5	125
5	25
5	5
	1

We see that 3375 = 3 × 3 × 3 × 5 × 5 × 5

Thus, $\sqrt[3]{3375} = \sqrt[3]{\underline{3 \times 3 \times 3} \times \underline{5 \times 5 \times 5}}$ = $3 \times 5 = 15$

Question 9: $\sqrt[3]{-\mathbf{216}}$

Solution - Since $\sqrt[3]{-216} = -\sqrt[3]{216}$

First we resolve 216 into prime factors:

2	216
2	108
2	54
3	27
3	9
3	3
	1

We see that 216 = 2 × 2 × 2 × 3 × 3 × 3

Thus, $\sqrt[3]{-216} = -\sqrt[3]{\underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3}}$

= $-2 \times 3 = -6$

Question 10: $\sqrt[3]{-\mathbf{512}}$

Solution - Since $\sqrt[3]{-512} = -\sqrt[3]{512}$

First we resolve 512 into prime factors:

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

We see that 512 = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Thus, $\sqrt[3]{-512} = -\sqrt[3]{\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}}$

= $-2 \times 2 \times 2 = -8$

Question 11: $\sqrt[3]{-\mathbf{1331}}$

Solution - Since $\sqrt[3]{-1331} = -\sqrt[3]{1331}$

First we resolve 1331 into prime factors:

11	1331
11	121
11	11
	1

We see that 1331 = $11 \times 11 \times 11$

Thus, $\sqrt[3]{-1331} = -\sqrt[3]{\underline{11 \times 11 \times 11}}$

$= -11$

Question 12: $\sqrt[3]{\mathbf{\frac{27}{64}}}$

Solution - Since $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$

Thus, $\sqrt[3]{\frac{27}{64}} = \frac{\sqrt[3]{27}}{\sqrt[3]{64}}$

We resolve 27 and 64 into prime factors

3	27
3	9
3	3
	1

2	64
2	32
2	16
2	8
2	4
2	2
	1

We see that 27 = 3 × 3 × 3

64 = 2 × 2 × 2 × 2 × 2 × 2

Thus, $\sqrt[3]{\frac{27}{64}} = \frac{\sqrt[3]{27}}{\sqrt[3]{64}} = \frac{\sqrt[3]{3\times3\times3}}{\sqrt[3]{2\times2\times2\times2\times2\times2}} = \frac{3}{2\times2} = \frac{3}{4}$

Question 13: $\sqrt[3]{\frac{\mathbf{125}}{\mathbf{216}}}$

Solution - Since $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$

Thus, $\sqrt[3]{\frac{125}{216}} = \frac{\sqrt[3]{125}}{\sqrt[3]{216}}$

We resolve 125 and 216 into prime factors

5	125
5	25
5	5
	1

2	216
2	108
2	54
3	27
3	9
3	3
	1

We see that 125 = 5 × 5 × 5

216 = 2 × 2 × 2 × 3 × 3 × 3

Thus, $\sqrt[3]{\frac{125}{216}} = \frac{\sqrt[3]{125}}{\sqrt[3]{216}} = \frac{\sqrt[3]{5\times5\times5}}{\sqrt[3]{2\times2\times2\times3\times3\times3}} = \frac{5}{2\times3} = \frac{5}{6}$

Question 14: $\sqrt[3]{\frac{-27}{125}}$

Solution - Since $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$

Thus, $\sqrt[3]{\frac{-27}{125}} = \frac{-\sqrt[3]{27}}{\sqrt[3]{125}}$

We resolve 125 and 27 into prime factors

5	125
5	25
5	5
	1

3	27
3	9
3	3
	1

We see that 125 = $5 \times 5 \times 5$

27 = $3 \times 3 \times 3$

Thus, $\sqrt[3]{\frac{-27}{125}} = \frac{-\sqrt[3]{27}}{\sqrt[3]{125}} = \frac{-\sqrt[3]{3\times3\times3}}{\sqrt[3]{5\times5\times5}} = \frac{-3}{5}$

Question 15: $\sqrt[3]{\frac{-64}{343}}$

Solution - Since $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$

Thus, $\sqrt[3]{\frac{-64}{343}} = \frac{-\sqrt[3]{64}}{\sqrt[3]{343}}$

We resolve 64 and 343 into prime factors

2	64
2	32
2	16
2	8
2	4
2	2
	1

7	343
7	49
7	7
	1

We see that 64 = $2 \times 2 \times 2 \times 2 \times 2 \times 2$

343 = $7 \times 7 \times 7$

Thus, $\sqrt[3]{\frac{-64}{343}} = \frac{-\sqrt[3]{64}}{\sqrt[3]{343}} = \frac{-\sqrt[3]{2\times2\times2\times2\times2\times2}}{\sqrt[3]{7\times7\times7}} = \frac{-(2\times2)}{7} = \frac{-4}{7}$

Question 16: $\sqrt[3]{\mathbf{64 \times 729}}$

Solution - Since $\sqrt[3]{ab} = (\sqrt[3]{a} \times \sqrt[3]{b})$

$\sqrt[3]{64 \times 729} = \sqrt[3]{64} \times \sqrt[3]{729}$

We resolve 64 and 729 into prime factors:

2	64
2	32
2	16
2	8
2	4
2	2
	1

3	729
3	243
3	81
3	27
3	9
3	3
	1

We can see 64 = $\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}$

$= 2^3 \times 2^3 = (2 \times 2)^3 = (4)^3$

729 = $3 \times 3 \times 3 \times 3 \times 3 \times 3$

$= 3^3 \times 3^3 = (3 \times 3)^3 = (9)^3$

Thus, $\sqrt[3]{64 \times 729} = \sqrt[3]{64} \times \sqrt[3]{729} = 4 \times 9 = 36$

Question 17: $\sqrt[3]{\frac{\mathbf{729}}{\mathbf{1000}}}$

Solution - Since $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$

Thus, $\sqrt[3]{\frac{729}{1000}} = \frac{\sqrt[3]{729}}{\sqrt[3]{1000}}$

We resolve 729 and 1000 into prime factors

3	729
3	243
3	81
3	27
3	9
3	3
	1

2	1000
2	500
2	250
5	125
5	25
5	5
	1

We see that 729 = 3 × 3 × 3 × 3 × 3 × 3

1000 = 2 × 2 × 2 × 5 × 5 × 5

Thus, $\sqrt[3]{\frac{729}{1000}} = \frac{\sqrt[3]{729}}{\sqrt[3]{1000}} = \frac{\sqrt[3]{3\times3\times3\times3\times3\times3}}{\sqrt[3]{2\times2\times2\times5\times5\times5}} = \frac{3\times3}{2\times5} = \frac{9}{10}$

Question 18: $\sqrt[3]{\frac{-512}{343}}$

Solution - Since $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$

Thus, $\sqrt[3]{\frac{-512}{343}} = \frac{-\sqrt[3]{512}}{\sqrt[3]{343}}$

We resolve 512 and 343 into prime factors

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

7	343
7	49
7	7
	1

We see that 512 = 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2 × 2

343 = 7 × 7 × 7

Thus, $\sqrt[3]{\frac{-512}{343}} = \frac{-\sqrt[3]{512}}{\sqrt[3]{343}} = \frac{-\sqrt[3]{2\times2\times2\times2\times2\times2\times2\times2\times2}}{\sqrt[3]{7\times7\times7}} = \frac{-(2\times2\times2)}{7} = \frac{-8}{7}$

Exercise 4D

Question 1 - Which of the following numbers is a perfect cube?

- **(a) 141**
- **(b) 294**
- **(c) 216**
- **(d) 496**

Solution (a) 141

We resolve 141 into prime factors

3	141
47	47
	1

$141 = 3 \times 47$

Since 141 cannot be expressed as a product of triplets

Hence, 141 is not a perfect cube.

(b) 294

We resolve 294 into prime factors

2	294
3	147
7	49
7	7
	1

$294 = 2 \times 3 \times 7 \times 7$

Since 294 cannot be expressed as a product of triplets

Hence, 294 is not a perfect cube.

(c) 216

We resolve 216 into prime factors

2	216
2	108
2	54
3	27
3	9
3	3
	1

$216 = 2 \times 2 \times 2 \times 3 \times 3 \times 3$

Since 216 can be expressed as a product of triplets.

Hence, 216 is a perfect cube.

(d) 496

We resolve 496 into prime factors

2	496
2	248
2	124
2	62
31	31
	1

$496 = 2 \times 2 \times 2 \times 2 \times 31$

Since 496 cannot be expressed as a product of triplets

Hence, 496 is not a perfect cube.

Question 2 - Which of the following numbers is a perfect cube?

(a) 1152
(b) 1331
(c) 2016
(d) 739

Solution (a) 1152

We resolve 1152 into prime factors

2	1152
2	576
2	288
2	144
2	72
2	36
2	18
3	9
3	3
	1

$1152 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3$

Since 1152 cannot be expressed as a product of triplets

Hence, 115 is not a perfect cube.

(b) 1331

We resolve 1331 into prime factors

11	1331
11	121
11	11
	1

$1331 = 11 \times 11 \times 11$

Since 1331 can be expressed as a product of triplets

Hence, 1331 is a perfect cube.

(c) 2016

We resolve 2016 into prime factors

2	2016
2	1008
2	504
2	252
2	126
3	63
3	21
7	7
	1

$2016 = 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 7$

Since2016 cannot be expressed as a product of triplets

Hence, 2016 is not a perfect cube.

(d) 739

Since 739 is a prime number.

$739 = 1 \times 739$

Since 739 cannot be expressed as a product of triplets

Hence, 739 is not a perfect cube.

Question 3: $\sqrt[3]{\mathbf{512}}$ =?

Solution - First we resolve 512 into prime factors:

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

We see that $512 = 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

Thus, $\sqrt[3]{512} = \sqrt[3]{\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2}}$

$= 2 \times 2 \times 2 = 8$

Question 4: $\sqrt[3]{\mathbf{125 \times 64}}$

Solution - Since $\sqrt[3]{ab} = (\sqrt[3]{a} \times \sqrt[3]{b})$

$\sqrt[3]{125 \times 64} = \sqrt[3]{8000}$

We resolve 8000 into prime factors:

2	8000
2	4000
2	2000
2	1000
2	500
2	250
5	125
5	25
5	5
	1

We can see 8000 = $\underline{2 \times 2 \times 2} \times \underline{2 \times 2 \times 2} \times \underline{5 \times 5 \times 5}$

$= 2^3 \times 2^3 \times 5^3 = (2 \times 2 \times 5)^3 = (20)^3$

Thus, $\sqrt[3]{8000} = \sqrt[3]{\underline{20 \times 20 \times 20}} = 20$

Question 5: $\sqrt[3]{\frac{64}{343}}$

Solution - Since $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$

Thus, $\sqrt[3]{\frac{64}{343}} = \frac{\sqrt[3]{64}}{\sqrt[3]{343}}$

We resolve 64 and 343 into prime factors

7	343
7	49
7	7
	1

2	64
2	32
2	16
2	8
2	4
2	2
	1

We see that 64 = $2 \times 2 \times 2 \times 2 \times 2 \times 2$

343 = $7 \times 7 \times 7$

Thus, $\sqrt[3]{\frac{64}{343}} = \frac{\sqrt[3]{64}}{\sqrt[3]{343}} = \frac{\sqrt[3]{2\times2\times2\times2\times2\times2}}{\sqrt[3]{7\times7\times7}} = \frac{(2\times2)}{7} = \frac{4}{7}$

Question 6: $\sqrt[3]{\frac{-512}{729}}$

Solution - Since $\sqrt[3]{\frac{a}{b}} = \frac{\sqrt[3]{a}}{\sqrt[3]{b}}$

Thus, $\sqrt[3]{\frac{-512}{729}} = \frac{-\sqrt[3]{512}}{\sqrt[3]{729}}$

We resolve 512 and 729 into prime factors

2	512
2	256
2	128
2	64
2	32
2	16
2	8
2	4
2	2
	1

3	729
3	243
3	81
3	27
3	9
3	3
	1

We see that 512 = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2$

729 = $3 \times 3 \times 3 \times 3 \times 3 \times 3$

Thus, $\sqrt[3]{\frac{-512}{729}} = \frac{-\sqrt[3]{512}}{\sqrt[3]{729}} = \frac{-\sqrt[3]{2\times2\times2\times2\times2\times2\times2\times2\times2}}{\sqrt[3]{3\times3\times3\times3\times3\times3}} = \frac{-(2\times2\times2)}{3\times3} = \frac{-8}{9}$

Question 7 - By what least number should 648 be multiplied to get a perfect cube?

Solution - First we resolve 648 into prime factors:

2	648
2	324
2	162
3	81
3	27
3	9
3	3
	1

We have, 648 = $\underline{2 \times 2 \times 2} \times \underline{3 \times 3 \times 3} \times 3$

Since it can be seen that 648 must be multiplied by $(3\times 3 = 9)$ to be a perfect cube

Question 8 - By what least number should 1536 be divided to get a perfect cube?

Solution - First we resolve 1536 into prime factors:

2	1536
2	768
2	384
2	192
2	96
2	48
2	24
2	12
2	6
3	3
	1

We have, 1536 = $2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3$

Since it can be seen that 1536 must be divided by 3 to be a perfect cube

=> 3 is the smallest number by which 1536 must be divided so that quotient is a perfect cube

Question 9: $(\mathbf{1\frac{3}{10}})^{\mathbf{3}}$ =?

Solution - Since $1\frac{3}{10} = \frac{13}{10}$

We have to find $(\frac{13}{10})^3$

$(\frac{13}{10})^3 = \frac{13\times13\times13}{10\times10\times10} = \frac{2197}{1000} = 2\frac{197}{1000}$

Question 10: $(\mathbf{0.8})^{\mathbf{3}}$ =?

Solution - Since $0.8 = \frac{8}{10}$

We have to find $(\frac{8}{10})^3$

$(\frac{8}{10})^3 = \frac{8\times8\times8}{10\times10\times10} = \frac{512}{1000}$= 0.512