Chapter 1

Integers

Introduction

Integers are a set of those numbers consisting of whole numbers and negative numbers. Integers can never be in the form of fraction or decimal. The examples of integers are 0. -1, 6, 3, -4...etc.

Integers include positive numbers, negative numbers and zero. Positive numbers are 1, 2, 3, 4…., Negative numbers are -1, -2, -3, -4….

Multiplication of Integers

While multiplying integers, we must keep in mind two rules which are as follows:

Rule1: The product of two integers carrying opposite signs is equal to the additive inverse of the product of their absolute values. For example: $5 \times (-4) = -20$

In the above example, 5 is positive integer and -4 is negative integer, the product of these two is equal to the additive inverse of$5 \times 4 = 20$, that is -20

Rule 2: The product of two integers carrying like signs is equal to the product of their absolute values. For example: $6 \times 3 = 18$ and $-4 \times -3 = 12$

In the above examples, both 6 and 3 carry positive sign thus the multiplication of these two is the product of their absolute values, that is 18. Also, in another example both 4 and 3 carry negative signs, thus their product will be 12.

Properties of multiplication:

Property 1: Closure property – For any two integers 'a' and 'b', we have $a \times b$ is also an integer.

Property 2: Commutative Property – It states that two integers can be multiplied in either order. That is, for any two integers 'a' and 'b', we have

$$a \times b = b \times a$$

Property 3: Associative Property – For any three integers ‘a’, ‘b’ and ‘c’, we have

$$(a \times b) \times c = a \times (b \times c)$$

Property 4: Distributive property - For any three integers ‘a’, ‘b’ and ‘c’, we have

(i) $a \times (b + c) = a \times b + a \times c$

(ii) $(b + c) \times a = b \times a + c \times a$

Property 5: Multiplicative identity – It states that when we multiply a number by 1, we get the same number. That is, for every integer ‘a’, we have

$$a \times 1 = a = 1 \times a$$

Property 6: Property of zero – For every integer ‘a’, we have

$$a \times 0 = 0 = 0 \times a$$

Property 7: For any integer ‘a’, we have

$$a \times (-1) = -a = (-1) \times a$$

Property 8: For any three integers ‘a’, ’b’ and ‘c’ such that a > b, we have

(i) $a \times c > b \times c$, if c is positive

(ii) $a \times c < b \times c$, if c is negative

Property 9: (i)$(-a1) \times (-a2) \times (-a3) \times \ldots \times (-an) = (a1 \times a2 \times a3 \times \ldots \times an)$, if n is even

(ii)$(-a1) \times (-a2) \times (-a3) \times \ldots (-an) = -(a1 \times a2 \times a3 \times \ldots \times an)$, if n is odd

(iii)$\underbrace{(-1) \times (-1) \times (-1) \times \ldots \times (-1)}_{\text{n - times}} = 1$, if n is even

(iv)$\underbrace{(-1) \times (-1) \times (-1) \times \ldots \times (-1)}_{\text{n - times}} = -1$, if n is odd

Examples

Example 1 – Find each of the following products:

(i) $(-115) \times 8$

Solution – We can write $(-115) \times 8 = -(115 \times 8) = -920$

Explanation: Here, 115 is negative integer and 8 is positive integer, the product of these two is equal to the additive inverse of$115 \times 8 = 920$, that is -920.

(ii) $9 \times (-3) \times (-6)$

Solution – By using properties of multiplication,

We can write $9 \times (-3) \times (-6) = 9 \times \{(-3) \times (-6)\} = 9 \times 18 = 162$

Explanation: Here, both 3 and 6 carry the negative sign, thus the multiplication of these two is equal is the product of their absolute values, that is 18

(iii) $(-12) \times (-13) \times (-5)$

Solution – By using properties of multiplication,

We can write $(-12) \times (-13) \times (-5) = (-12) \times \{(-13) \times (-5)\} = (-12) \times 65 = -780$

Explanation: Here, both 13 and 5 carry the negative sign, thus the multiplication of these two is the product of their absolute values, that is 65. Now, 12 is negative number and 65 is positive number, the product of these two is equal to the additive inverse of$12 \times 65 = 780$, that is -780.

Example 2 – Evaluate each of the following products:

(i) $(-1) \times (-2) \times (-3) \times (-4) \times (-5)$

Solution – Using the property of multiplication,$(-a1) \times (-a2) \times (-a3) \times \ldots (-an) = -(a1 \times a2 \times a3 \times \ldots \times an)$, if n is odd

Here, n = 5, which is odd

We have $(-1) \times (-2) \times (-3) \times (-4) \times (-5)$

$= -(1 \times 2 \times 3 \times 4 \times 5)$

$= -(6 \times 4 \times 5)$

$= -(6 \times 20)$

$= -120$

(ii) $(-3) \times (-6) \times (-9) \times (-12)$

Solution – Using the property of multiplication, $(-a1) \times (-a2) \times (-a3) \times \ldots (-an) = (a1 \times a2 \times a3 \times \ldots \times an)$, if n is even

Here, n = 4, which is even

We have $(-3) \times (-6) \times (-9) \times (-12)$

$= (3 \times 6 \times 9 \times 12)$

$= (18 \times 9 \times 12)$

$= (18 \times 108)$

$= 1944$

(iii) $(-1) \times (-1) \times (-1) \times \ldots$ **50** $\boldsymbol{times}$

Solution – Using the property of multiplication, $(-1) \times (-1) \times (-1) \times \ldots \times (-1) = 1$, if n is even

Here, n = 50, which is even

We have $(-1) \times (-1) \times (-1) \times \ldots 50$ times $= 1$

(iv) $(-1) \times (-1) \times (-1) \times \ldots$ **151** $\boldsymbol{times}$

Solution – Using the property of multiplication, $(-1) \times (-1) \times (-1) \times \ldots \times (-1) = -1$, if n is odd

Here, n = 151, which is odd

We have $(-1) \times (-1) \times (-1) \times \ldots 151 \text{ times} = -1$

Example 3 – Find the value of

(i) $15625 \times (-2) + (-15625) \times 98$

Solution – Using the distributive property of multiplication,

$$a \times (b + c) = a \times b + a \times c$$

We have, $15625 \times (-2) + (-15625) \times 98 = (-15625) \times 2 + (-15625) \times 98 = (-15625) \times (2 + 98) = (-15625) \times 100 = -1562500$

(ii) $18946 \times 99 - (-18946)$

Solution – We can write $18946 \times 99 - (-18946) = 18946 \times 99 + 18946$

Since, $18946 = 18946 \times 1$

$= 18946 \times 99 + 18946 \times 1$

Now, by using distributive property,

$a \times (b + c) = a \times b + a \times c$

We have, $18946 \times 99 + 18946 \times 1$

$=18946 \times (99 + 1) = 18946 \times 100 = 1894600$

(iii) $1569 \times 887 - 569 \times 887$

Solution - Since $(b - c) \times a = b \times a - c \times a$

Thus, we have $1569 \times 887 - 569 \times 887$

$= (1569 - 569) \times 887$

$= 1000 \times 887 = 887000$

Exercise 1.1

Question 1 – Determine each of the following products:

(i) 12×7

Solution – $12 \times 7 = 84$

Explanation: Here**,** both 12 and 7 carry positive sign thus the multiplication of these two is the product of their absolute values, that is 84

(ii) $(-15) \times 8$

Solution: $(-15) \times 8 = -(15 \times 8) = -120$

Explanation: Here, -15 is negative integer and 8 is positive integer, the product of these two is equal to the additive inverse of$15 \times 8 = 120$, that is -120.

(iii) $(-25) \times (-9)$

Solution: $(-25) \times (-9) = (25 \times 9) = 225$

Explanation: Here, both 25 and 9 carry the negative sign, thus the multiplication of these two is equal is the product of their absolute values, that is 225

(iv) $125 \times (-8)$

Solution: $125 \times (-8) = -(125 \times 8) = -1000$

Explanation: Here, 125 is positive integer and -8 is negative integer, the product of these two is equal to the additive inverse of$125 \times 8 = 1000$, that is -1000.

Question 2 – Find each of the following products:

(i) $3 \times (-8) \times 5$

Solution – By using properties of multiplication,

We can write $3 \times (-8) \times (5) = 3 \times \{(-8) \times 5\} = 3 \times (-40) = -120$

Explanation: Here, -8 is negative and 5 is positive, thus the multiplication of these two is equal is additive inverse of$8 \times 5 = 40$, that is -40. Again, 3 is Positive and -40 is negative, thus the multiplication of these two is equal is additive inverse of$3 \times 40 = 120$, that is -120

(ii) $\mathbf{9 \times (-3) \times (-6)}$

Solution – By using properties of multiplication,

We can write $9 \times (-3) \times (-6) = 9 \times \{(-3) \times (-6)\} = 9 \times 18 = 162$

Explanation: Here, both 3 and 6 carry the negative sign, thus the multiplication of these two is equal is the product of their absolute values, that is 18

(iii) $\mathbf{(-2) \times 36 \times (-5)}$

Solution – By using properties of multiplication,

We can write $(-2) \times 36 \times (-5) = (-2) \times \{(36) \times (-5)\} = (-2) \times (-180) = 360$

Explanation: Here, 36 is positive and -5 is negative, thus the multiplication of these two is equal is additive inverse of$36 \times 5 = 180$, that is -180. Now, -2 is negative and -180 is also negative, thus the multiplication of these two is equal is the product of their absolute values, that is 360

(iv) $\mathbf{(-2) \times (-4) \times (-6) \times (-8)}$

Solution – Using the property of multiplication,$(-a1) \times (-a2) \times (-a3) \times \ldots (-an) = (\, a1 \times a2 \times a3 \times \ldots \times an)$, if n is even

Here, n = 4, which is even

We have $(-2) \times (-4) \times (-6) \times (-8)$

$= (2 \times 4 \times 6 \times 8)$

$= (8 \times 6 \times 8) = (48 \times 8) = 384$

Question 3 – Find the value of:

(i) $\mathbf{1487 \times 327 + (-487) \times 327}$

Solution – Using the distributive property of multiplication,

$$(b - c) \times a = b \times a - c \times a$$

We have, $1487 \times 327 + (-487) \times 327 = 1487 \times 327 - 487 \times 327 = (1487 - 487) \times 327 = 1000 \times 327 = 327000$

(ii) $28945 \times 99 - (-28945)$

Solution – We can write $28945 \times 99 - (-28945) = 28945 \times 99 + 28945$

Since, $28945 = 28945 \times 1$

$= 28945 \times 99 + 28945 \times 1$

Now, by using distributive property,

$a \times (b + c) = a \times b + a \times c$

We have, $28945 \times 99 + 28945 \times 1$

$=28945 \times (99 + 1) = 28945 \times 100 = 2894500$

Question 4 – Complete the following multiplication table:

Second Number

x	**-4**	**-3**	**-2**	**-1**	**0**	**1**	**2**	**3**	**4**
-4									
-3									
-2									
-1									
0									
1									
2									
3									
4									

First Number

Is the multiplication table symmetrical about the diagonal joining the upper left corner to the lower right corner?

Solution – While solving this table, we keep in mind the 2 rules as follows:

(1) The product of two integers carrying opposite signs is equal to the additive inverse of the product of their absolute values.

(2) The product of two integers carrying like signs is equal to the product of their absolute values.

Second Number

First Number

x	-4	-3	-2	-1	0	1	2	3	4
-4	16	12	8	4	0	-4	-8	-12	-16
-3	12	9	6	3	0	-3	-6	-9	-12
-2	8	6	4	2	0	-2	-4	-6	-8
-1	4	3	2	1	0	-1	-2	-3	-4
0	0	0	0	0	0	0	0	0	0
1	-4	-3	-2	-1	0	1	2	3	4
2	-8	-6	-4	-2	0	2	4	6	8
3	-12	-9	-6	-3	0	3	6	9	12
4	-16	-12	-8	-4	0	4	8	12	16

From the above table, we can see that given table is symmetrical about the diagonal joining the upper left corner to the lower right corner**.**

Question 5 – Determine the integer whose product with '-1' is

(a) 58

Solution – We know that the product of two integers carrying like signs is equal to the product of their absolute values.

So, when we multiply 58 by (-1), we get

$58 \times (-1) = -58$

Thus, -58 is the integer whose product with -1 is 58

(ii) 0

Solution - We know that the product of two integers carrying like signs is equal to the product of their absolute values.

So, when we multiply 0 by (-1), we get

$0 \times (-1) = -0 = 0$

Thus, 0 is the integer whose product with -1 is 0

(iii) -225

Solution - The product of two integers carrying like signs is equal to the product of their absolute values.

So, when we multiply -225 by (-1), we get

$(-225) \times (-1) = 225$

Thus, 225 is the integer whose product with -1 is -225

Question 6 – What will be the sign of the product if we multiply together

(i) 8 negative integers and 1 positive integer?

Solution – We know that,$(-a1) \times (-a2) \times (-a3) \times \ldots (-an) = (\, a1 \times a2 \times a3 \times \ldots \times an)$, if n is even.

Thus, product of 8 negative integers is positive.

Therefore, when we multiply 8 negative integers with 1 positive integer, the sign of the product will be positive as $(+ve) \times (+ve) = (+ve)$

(ii) 21 negative integers and 1 positive integer?

Solution - We know that,$(-a1) \times (-a2) \times (-a3) \times \ldots (-an) = -(\, a1 \times a2 \times a3 \times \ldots \times an)$, if n is odd

Thus, product of 21 negative integers is negative.

Therefore, when we multiply 21 negative integers with 1 positive integer, the sign of the product will be negative as $(+ve) \times (-ve) = (-ve)$

(iii) 199 negative integers and 10 positive integers

Solution - We know that,$(-a1) \times (-a2) \times (-a3) \times \ldots (-an) = -(\, a1 \times a2 \times a3 \times \ldots \times an)$, if n is odd

Thus, product of 21 negative integers is negative and product of 10 positive integers is positive.

Therefore, when we multiply 199 negative integers with 10 positive integers, the sign of the product will be negative as $(+ve) \times (-ve) = (-ve)$

Question 7 – State which is greater:

(i) $\mathbf{(8 + 9) \times 10}$ ***and*** $\mathbf{8 + 9 \times 10}$

Solution – We use BODMAS for solving this

1) $(8 + 9) \times 10 = 17 \times 10 = 170$

2) $8 + 9 \times 10 = 8 + 90 = 98$

Clearly, 170 > 98

Therefore, $(8 + 9) \times 10$ is greater than $8 + 9 \times 10$

(ii) $\mathbf{(8 - 9) \times 10}$ ***and*** $\mathbf{8 - 9 \times 10}$

Solution - We use BODMAS for solving this

1) $(8 - 9) \times 10 = -1 \times 10 = -10$

2) $8 - 9 \times 10 = 8 - 90 = -82$

Clearly, -10 > -82

Therefore, $(8 - 9) \times 10$ is greater than $8 - 9 \times 10$

(iii) $\{(-2) - 5\} \times (-6) \boldsymbol{and} (-2) - 5 \times (-6)$

Solution - We use BODMAS for solving this

1) $\{(-2) - 5\} \times (-6) = \{-2 - 5\} \times (-6) = -7 \times (-6) = (7 \times 6) = 42$

2)$(-2) - 5 \times (-6) = (-2) - (-30) = -2 + 30 = 28$

Clearly, 42 > 28

Therefore, $\{(-2) - 5\} \times (-6)$is greater than $(-2) - 5 \times (-6)$

Question 8 – (i) If$a \times (-1) = -30$, is the integer 'a' positive or negative?

Solution – Since the product of two integers carrying opposite signs is equal to the additive inverse of the product of their absolute values.

Thus, if$a \times (-1) = -30$, then 'a' must be positive integer

(ii) If$a \times (-1) = 30$, is the integer 'a' positive or negative?

Solution – Since the product of two integers carrying like signs is equal to the product of their absolute values.

Thus, if$a \times (-1) = 30$, then 'a' must be negative integer

Question 9 – Verify the following:

(i) $19 \times \{7 + (-3)\} = 19 \times 7 + 19 \times (-3)$

Solution – LHS: $19 \times \{7 + (-3)\}$

= $19 \times \{7 - 3\}$

$= 19 \times 4 = 76$

RHS: $19 \times 7 + 19 \times (-3)$

$= 133 + (-57)$

$= 133 - 57 = 76$

Thus, LHS = RHS

Hence verified

(ii) $\mathbf{(-23)\{(-5) + (+19)\} = (-23) \times (-5) + (-23) \times (+19)}$

Solution – LHS: $(-23)\{(-5) + (+19)\}$

$= (-23)\{-5 + 19\}$

$= (-23)(14)$

$= -322$

RHS: $(-23) \times (-5) + (-23) \times (+19)$

$= (23 \times 5) + (-)(23 \times 19)$

$= 115 - 437$

$= -322$

Thus, LHS = RHS

Hence verified

Question 10 – Which of the following statements are true?

(i) The product of a positive and a negative integer is negative.

Solution – True

Explanation: The product of two integers carrying opposite signs is equal to the additive inverse of the product of their absolute values.

(ii) The product of three negative integers is a negative integer.

Solution – True

Explanation: Here n = 3 which is odd and we know that the product of odd number of negative integers is always negative.

(iii) Of the two integers, if one is negative, then their product must be positive.

Solution – False

Explanation: Their product will be positive only if sign of other integer is negative. However if the sign of other integer is positive then their product will be negative.

(iv)For all non-zero integers 'a' and 'b', $a \times b$ is always greater than either 'a' or 'b'.

Solution – False

Explanation: If we take a = 2 and b = -2

Then, $2 \times (-2) = -4$

But, -4 is not greater than a = 2 or b = -2

(v) The product of a negative and a positive integer may be zero.

Solution – False

Explanation: Let us take a negative number be -5 and a positive number be 2

Then if we multiply $(-5) \times 2 = -10$

Thus, we find that the product of a negative and a positive integer is always a negative integer.

(vi)There does not exist an integer b such that for a > 1, $a \times b = b \times a = b$

Solution – True

Explanation: If a = 2, then $2 \times b = b \times 2 = 2b \neq b$

Thus, it is not possible for any value of a greater than 1.

Division of Integers

Let us have a look on some terms,

Consider two integers 'a' and 'b'

(1) Dividend (a): It is the number to be divided.

(2) Divisor (b): It is the number which divides.

(3) Quotient: it is the result when we divide a by b

While dividing the integers, we must keep in mind the following rules

Rule 1: The division of two integers carrying like signs is equal to the division of their absolute values. It means that if two integers are of same sign, then the quotient is always positive integer

Rule 2: The division of two integers carrying opposite signs always results in a negative integer.

Properties of Division:

- If 'a' and 'b' are integers, then $a \div b$ is not necessarily an integer
- If 'a' is an integer other than 0, then $a \div a = 1$
- For each integer 'a', we have $a \div 1 = 1$
- If $a \neq 0$, then $0 \div a = 0$
- If 'a' is an integer, then $a \div 0$ is not defined
- If 'a', 'b' and 'c' are non-zero integers, then $(a \div b) \div c = a \div (b \div c)$, unless c =1
- If 'a', 'b' and 'c' are integers, then

(i) $a > b => a \div c > b \div c$, if c is positive

(ii) $a > b => a \div c < b \div c$, if c is negative

Examples

Example 1 – Divide

(i) 84 by 7

Solution – We will find $84 \div 7$

While dividing two integers carrying like signs, we will divide their absolute values and put a plus sign to the quotient.

We can rewrite it as $\frac{84}{7} = 12$

(ii) -91 by 13

Solution – We will find $-91 \div 13$

While dividing two integers carrying unlike signs, we will divide their absolute values and put a minus sign to the quotient.

We have, $-\frac{91}{13} = -7$

(iii) -98 by -14

Solution – We will find $-98 \div (-14)$

While dividing two integers carrying like signs, we will divide their absolute values and put a plus sign to the quotient.

We can rewrite it as $\frac{-98}{-14} = \frac{98}{14} = 7$

(iv) 324 by -27

Solution – We will find $324 \div (-27)$

While dividing two integers carrying unlike signs, we will divide their absolute values and put a minus sign to the quotient.

We have, $-\frac{324}{27} = -12$

Example 2 – Find the quotient in each of the following:

(i) $\mathbf{(-1728) \div 12}$

Solution – We will find $-1728 \div 12$

While dividing two integers carrying unlike signs, we will divide their absolute values and put a minus sign to the quotient.

We have, $-\frac{1728}{12} = -144$

(ii) $\mathbf{(-15625) \div (-125)}$

Solution – We will find $(-15625) \div (-125)$

While dividing two integers carrying like signs, we will divide their absolute values and put a plus sign to the quotient.

We can rewrite it as $\frac{-15625}{-125} = \frac{15625}{125} = 125$

(iii) $\mathbf{30000 \div (-100)}$

Solution - We will find $30000 \div (-100)$

While dividing two integers carrying unlike signs, we will divide their absolute values and put a minus sign to the quotient.

We have, $-\frac{30000}{100} = -300$

Example 3 – Find the value of

(i) $\mathbf{[32 + 2 \times 17 + (-6)] \div 15}$

Solution – We will solve this using BODMAS rule as follows:

We have, $[32 + 2 \times 17 + (-6)] \div 15$

=> $[32 + 34 + (-6)] \div 15$

=> $[66 - 6] \div 15$

=> $60 \div 15 = \frac{60}{15} = 4$

(ii) $\mathbf{\left||-17| + 17\right| \div \left||-25| - 42\right|}$

Solution – We know that $|-a| = a$

We have, $\left||-17| + 17\right| \div \left||-25| - 42\right|$

=> $|17 + 17| \div |25 - 42|$

=> $|34| \div |-17| = 34 \div 17 = \frac{34}{17} = 2$

Example 4 – Simplify: $\mathbf{\{36 \div (-9)\} \div \{(-24) \div 6\}}$

Solution**:** While dividing two integers carrying unlike signs, we will divide their absolute values and put a minus sign to the quotient.

$\left\{-\frac{36}{9}\right\} \div \{-\frac{24}{6}\}$

=> $(-4) \div (-4)$

While dividing two integers carrying like signs, we will divide their absolute values and put a plus sign to the quotient.

=> $\frac{4}{4} = 1$

Exercise 1.2

Question 1 - Divide:

(i) 102 by 17

Solution - We will find $102 \div 17$

While dividing two integers carrying like signs, we will divide their absolute values and put a plus sign to the quotient.

We can rewrite it as $\frac{102}{17} = 6$

(ii) -85 by 5

Solution – We will find $-85 \div 5$

While dividing two integers carrying unlike signs, we will divide their absolute values and put a minus sign to the quotient.

We have, $-\frac{85}{5} = -17$

(iii) -161 by -23

Solution - We will find $-161 \div (-23)$

While dividing two integers carrying like signs, we will divide their absolute values and put a plus sign to the quotient.

We can rewrite it as $\frac{-161}{-23} = \frac{161}{23} = 7$

(iv) 76 by -19

Solution – We will find $76 \div (-19)$

While dividing two integers carrying unlike signs, we will divide their absolute values and put a minus sign to the quotient.

We have, $-\frac{76}{19} = -4$

(v) 17654 by -17654

Solution - We will find $17654 \div (-17654)$

While dividing two integers carrying unlike signs, we will divide their absolute values and put a minus sign to the quotient.

We have, $-\frac{17654}{17654} = -1$

(vi) (-729) by (-27)

Solution - We will find $(-729) \div (-27)$

While dividing two integers carrying like signs, we will divide their absolute values and put a plus sign to the quotient.

We can rewrite it as $\frac{-729}{-27} = \frac{729}{27} = 27$

(vii) 21590 by -10

Solution - We will find $21590 \div (-10)$

While dividing two integers carrying unlike signs, we will divide their absolute values and put a minus sign to the quotient.

We have, $-\frac{21590}{10} = -2159$

(viii) 0 by -135

Solution - We will find $0 \div (-135)$

While dividing two integers carrying unlike signs, we will divide their absolute values and put a minus sign to the quotient.

We have, $-\frac{0}{135} = 0$

Question 2 – Fill in the blanks:

(i) $296 \div \ldots = -148$

Solution – Let the blank space be 'x'

Then, we have $296 \div x = -148$

=> $\frac{296}{x} = -148$

By cross multiplication, we have,

$296 = -148x$

=> $x = \frac{296}{-148} = -\frac{296}{148} = -2$

Thus, $296 \div -2 = -148$

(ii) $-88 \div \ldots = 11$

Solution - Let the blank space be 'x'

Then, we have $-88 \div x = 11$

=> $\frac{-88}{x} = 11$

By cross multiplication, we have,

$-88 = 11x$

=> $x = \frac{-88}{11} = -\frac{88}{11} = -8$

Thus, $-88 \div -8 = 11$

(iii) $84 \div \ldots = 12$

Solution – Let the blank space be 'x'

Then, we have $84 \div x = 12$

$\Rightarrow \frac{84}{x} = 12$

By cross multiplication, we have,

$84 = 12x$

$\Rightarrow x = \frac{84}{12} = 7$

Thus, $84 \div 7 = 12$

(iv) ... $\div -5 = 25$

Solution – Let the blank space be 'x'

Then, we have $x \div (-5) = 25$

$\Rightarrow \frac{x}{-5} = 25$

By cross multiplication, we have

$x = 25 \times (-5) = -125$

Thus, $-125 \div (-5) = 25$

(v) ... $\div 156 = -2$

Solution – Let the blank space be 'x'

Then, we have $x \div 156 = -2$

$\Rightarrow \frac{x}{156} = -2$

By cross multiplication, we have

$x = 156 \times (-2) = -312$

Thus, $-312 \div 156 = -2$

Question 3 – Which of the following statements are true?

(i) $0 \div 4 = 0$

Solution – True

Explanation: When 0 is divided by any integer, the result is always 0.

(ii) $0 \div (-7) = 0$

Solution – True

Explanation: When 0 is divided by any integer, the result is always 0.

(iii) $-15 \div 0 = 0$

Solution – False

Explanation: When any integer is divided by 0 then the result is not defined. Thus, denominator should not be 0 in dividing two integers.

(iv) $0 \div 0 = 0$

Solution – False

Explanation: When any integer is divided by 0 then the result is not defined. Thus, the denominator should not be 0 in dividing two integers.

(v) $(-8) \div (-1) = -8$

Solution – False

Explanation: We know that while dividing two integers carrying like signs, we will divide their absolute values and put a plus sign to the quotient. Thus, we have $\frac{-8}{-1} = \frac{8}{1} = 8$. We get 8 as the quotient.

(vi) $-8 \div (-2) = 4$

Solution – True

Explanation: We know that while dividing two integers carrying like signs, we will divide their absolute values and put a plus sign to the quotient. Thus, we have $\frac{-8}{-2} = \frac{8}{2} = 4$. We get 4 as the quotient which is given to be true.

Operator precedence

While solving an expression consisting of many operations, we solve it by using a proper order 'DMAS' from left to right, where D = Division, M = Multiplication, A = Addition, S = Subtraction.

Examples:

Example 1 – Simplify: $24 - 4 \div 2 \times 3$

Solution: Using 'DMAS' rule, we have

$24 - 4 \div 2 \times 3$

$= 24 - 2 \times 3$ (Carrying out division)

$= 24 - 6$ (Carrying out Multiplication)

$= 18$ (Carrying out Subtraction)

Example 2 – Simplify: $(-20) + (-8) \div (-2) \times 3$

Solution - Using 'DMAS' rule, we have

$(-20) + (-8) \div (-2) \times 3$

$= -20 + (\frac{-8}{-2}) \times 3$

$= -20 + 4 \times 3$ (Carrying out division)

$= -20 + 12$ (Carrying out Multiplication)

$= -8$ (Carrying out Subtraction)

Example 3 – Simplify: $(-5) - (-48) \div (-16) + (-2) \times 6$

Solution - Using 'DMAS' rule, we have

$(-5) - (-48) \div (-16) + (-2) \times 6$

$= (-5) - \left(\frac{-48}{-16}\right) + (-2) \times 6$

$= (-5) - 3 + (-2) \times 6$ (Carrying out division)

$= (-5) - 3 + (-12)$ (Carrying out Multiplication)

$= -5 - 3 - 12$ (Carrying out addition)

$= -20$ (when all integers are negative, we simply add all integers and put negative sign to it)

Exercise 1.3

Find the value of

Question 1: $36 \div 6 + 3$

Solution: Using 'DMAS' rule, we have

$36 \div 6 + 3$

$= 6 + 3$ (Carrying out division)

$= 9$ (Carrying out Addition)

Question 2: $24 + 15 \div 3$

Solution: Using 'DMAS' rule, we have

$24 + 15 \div 3$

$= 24 + 5$ (Carrying out division)

$= 29$ (Carrying out Addition)

Question 3: $120 - 20 \div 4$

Solution: Using 'DMAS' rule, we have

$120 - 20 \div 4$

$= 120 - 5$ (Carrying out division)

$= 115$ (Carrying out Subtraction)

Question 4: $32 - (3 \times 5) + 4$

Solution: Using 'DMAS' rule, we have

$32 - (3 \times 5) + 4$

$= 32 - 15 + 4$ (Carrying out Multiplication)

$= 36 - 15$ (Carrying out Addition)

$= 21$ **(**Carrying out Subtraction)

Question 5: $3 - (5 - 6 \div 3)$

Solution: Using 'DMAS' rule, we have

$3 - (5 - 6 \div 3)$

$= 3 - (5 - 2)$ (Carrying out Division)

$= 3 - 3$ (Carrying out Subtraction)

$= 0$ **(**Carrying out Subtraction)

Question 6: $21 - 12 \div 3 \times 2$

Solution: Using 'DMAS' rule, we have

$21 - 12 \div 3 \times 2$

$= 21 - 4 \times 2$ (Carrying out Division)

$= 21 - 8$ (Carrying out Multiplication)

$= 13$ **(**Carrying out Subtraction)

Question 7: $16 + 8 \div 4 - 2 \times 3$

Solution: Using 'DMAS' rule, we have

$16 + 8 \div 4 - 2 \times 3$

$= 16 + 2 - 2 \times 3$ (Carrying out Division)

$= 16 + 2 - 6$ (Carrying out Multiplication)

$= 18 - 6$ (Carrying out Addition)

$= 12$ (Carrying out Subtraction)

Question 8: $28 - 5 \times 6 + 2$

Solution: Using 'DMAS' rule, we have

$28 - 5 \times 6 + 2$

$= 28 - 30 + 2$ (Carrying out Multiplication)

$= 30 - 30$ (Carrying out Addition)

$= 0$ **(**Carrying out Subtraction)

Question 9: $(-20) \times (-1) + (28) \div 7$

Solution: Using 'DMAS' rule, we have

$(-20) \times (-1) + (-28) \div 7$

$= -20 \times (-1) + (-4)$ (Carrying out Division)

$= 20 + (-4)$ (Carrying out Multiplication)

$= 20 - 4$

= 16 **(**Carrying out Subtraction)

Question 10: $\mathbf{(-2) + (-8) \div (-4)}$

Solution: Using 'DMAS' rule, we have

$(-2) + (-8) \div (-4)$

$= -2 + (\frac{-8}{-4})$

$= -2 + 2$ (Carrying out Division)

= 0 (Carrying out Subtraction)

Question 11: $\mathbf{(-15) + 4 \div (5 - 3)}$

Solution: Using 'DMAS' rule, we have

$(-15) + 4 \div (5 - 3)$

$= (-15) + 4 \div 2$ (Solving Subtraction in bracket)

$= -15 + 2$ (Carrying out Division)

$= -13$ (Carrying out Subtraction)

Question 12: $\mathbf{(-40) \times (-1) + (-28) \div 7}$

Solution: Using 'DMAS' rule, we have

$(-40) \times (-1) + (-28) \div 7$

$= -40 \times (-1) + (\frac{-28}{7})$

$= -40 \times (-1) + (-4)$ (Carrying out Division)

$= 40 + (-4)$ (Carrying out Multiplication)

$= 40 - 4$

= 36 (Carrying out Subtraction)

Question 13: $(-3) + (-8) \div (-4) - 2 \times (-2)$

Solution: Using 'DMAS' rule, we have

$(-3) + (-8) \div (-4) - 2 \times (-2)$

$= (-3) + \left(\frac{-8}{-4}\right) - 2 \times (-2)$

$= (-3) + 2 - 2 \times (-2)$ (Carrying out Division)

$= (-3) + 2 - (-4)$ (Carrying out Multiplication)

$= -3 + 2 + 4$

$= -3 + 6$ (Carrying out Addition)

= 3 (Carrying out Subtraction)

Question 14: $(-3) \times (-4) \div (-2) + (-1)$

Solution: Using 'DMAS' rule, we have

$(-3) \times (-4) \div (-2) + (-1)$

$= (-3) \times \left(\frac{-4}{-2}\right) + (-1)$

$= (-3) \times 2 + (-1)$ (Carrying out Division)

$= -6 + (-1)$ (Carrying out Multiplication)

$= -6 - 1$

$= -7$ (Carrying out Addition)

Use of Brackets

In the complex expression containing brackets within brackets, we will follow a particular order to solve it.

There are three types of brackets:

1) Common brackets = ( )

2) Curly brackets = { }

3) Square brackets = [ ]

In any expression, they are in the form [{( )}]

We first solve the innermost brackets ( ), then { } and then [ ] in any expression

Procedure for solving an expression

Step 1: See whether the given expression contains a vinculum or not. If a vinculum is present, then perform operations under it. Otherwise go to next step.

Step 2: see the innermost bracket and perform operations within it.

Step 3: Remove the innermost bracket by using following rules:

Rule1: If a bracket is preceded by a plus sign, remove it by writing its terms as they are.

Rule 2: If a bracket is preceded by a minus sign, change positive signs within it to negative and vice-versa.

Rule 3: If there is no sign between a number and a grouping symbol, then it means multiplication

Rule 4: If there is a number before some brackets then we multiply the number inside the brackets with the number outside the brackets.

Step 4: see the next innermost bracket and perform operations within it. Remove the second innermost bracket by using the rules in step 3. Continue this till all the brackets are removed.

Examples

Example 1 – Simplify: $27 - [5 + \{28 - (29 - 7)\}]$

Solution: Removing innermost bracket, we get

$= 27 - [5 + \{28 - 22\}]$

Now, removing second innermost bracket, we get

$= 27 - [5 + 6]$

Removing square brackets, we get

$= 27 - 11$

$= 16$

Example 2 - Simplify: $48 - [18 - \{16 - (5 - \overline{4 - 1})\}]$

Solution: Here, given expression contains vinculum. So, we first perform operations under it as follows:

$= 48 - [18 - \{16 - (5 - 3)\}]$

Removing innermost bracket, we get'

$= 48 - [18 - \{16 - 2\}]$

Removing curly brackets, we get

$= 48 - [18 - 14]$

Removing square brackets, we get

$= 48 - 4$

$= 44$

Example 3 – Simplify: $\mathbf{222 - [\frac{1}{3}\{42 + (56 - \overline{8 + 9})\} + 108]}$

Solution: Here, given expression contains vinculum. So, we first perform operations under it as follows:

$= 222 - [\frac{1}{3}\{42 + (56 - 17)\} + 108]$

Removing innermost bracket, we get'

$= 222 - [\frac{1}{3}\{42 + 39\} + 108]$

Removing curly brackets, we get

$= 222 - [\frac{1}{3} \times 81 + 108]$

$= 222 - [27 + 108]$

Removing square brackets, we get

$= 222 - 135$

$= 87$

Example 4 – Simplify: $\mathbf{39 - [23 - \{29 - (17 - \overline{9 - 3})\}]}$

Solution: Here, given expression contains vinculum. So, we first perform operations under it as follows:

$= 39 - [23 - \{29 - (17 - 6)\}]$

Removing innermost bracket, we get'

$= 39 - [23 - \{29 - 11\}]$

Removing curly brackets, we get

$= 39 - [23 - 18]$

Removing square brackets, we get

$= 39 - 5$

$= 34$

Example 5 – Simplify: $\mathbf{118 - [121 \div (11 \times 11) - (-4) - \{3 - \overline{9 - 2}\}]}$

Solution: Here, given expression contains vinculum. So, we first perform operations under it as follows:

$= 118 - [121 \div (11 \times 11) - (-4) - \{3 - 7\}]$

Removing common brackets, we get

$= 118 - [121 \div 121 - (-4) - \{3 - 7\}]$

Removing curly brackets, we get

$= 118 - [121 \div 121 - (-4) - (-4)]$

Performing division, we get

$= 118 - [1 - (-4) - (-4)]$

$= 118 - [1 + 4 + 4]$

Performing addition and removing square brackets, we get

$= 118 - 9$

$= 109$

Exercise 1.4

Question 1: $3 - (5 - 6 \div 3)$

Solution: Performing division in common brackets, we get

$= 3 - (5 - 2)$

Removing brackets, we get

$= 3 - 3$

$= 0$

Question 2: $-25 + 14 \div (5 - 3)$

Solution: Removing common brackets, we get

$= -25 + 14 \div 2$

Performing division, we get

$= -25 + 7$

Performing subtraction, we get

= -18

Question 3: $25 - \frac{1}{2}\{5 + 4 - (3 + 2 - \overline{1 + 3})\}$

Solution: Here, given expression contains vinculum. So, we first perform operations under it as follows:

$= 25 - \frac{1}{2}\{5 + 4 - (3 + 2 - 4)\}$

Removing innermost brackets, we get

$= 25 - \frac{1}{2}\{5 + 4 - 1\}$

Removing curly brackets, we get

$= 25 - \frac{1}{2} \times 8$

Performing Multiplication, we get

$= 25 - 4$

$= 21$

Question 4: $27 - [38 - \{46 - (15 - \overline{13 - 2})\}]$

Solution: Here, given expression contains vinculum. So, we first perform operations under it as follows:

$= 27 - [38 - \{46 - (15 - 11)\}]$

Removing innermost brackets, we get

$= 27 - [38 - \{46 - 4\}]$

Removing curly brackets, we get

$= 27 - [38 - 42]$

$= 27 - [-4]$

$= 27 + 4$

$= 31$

Question 5: $36 - [18 - \{14 - (15 - 4 \div 2 \times 2)\}]$

Solution: Removing innermost brackets, we get

$= 36 - [18 - \{14 - (15 - 2 \times 2)\}]$ (Performing division)

$= 36 - [18 - \{14 - (15 - 4)\}]$ (Performing multiplication)

$= 36 - [18 - \{14 - 11\}]$ (Performing subtraction)

Removing curly brackets, we get

$= 36 - [18 - 3]$

$= 36 - 15$

$= 21$

Question 6: $45 - [38 - \{60 \div 3 - (6 - 9 \div 3) \div 3\}]$

Solution: Removing innermost brackets, we get

$= 45 - [38 - \{60 \div 3 - (6 - 3) \div 3\}]$ (Performing division)

$= 45 - [38 - \{60 \div 3 - 3 \div 3\}]$

Removing curly brackets, we get

$= 45 - [38 - \{20 - 1\}]$ (Performing division)

$= 45 - [38 - 19]$

$= 45 - 19$

$= 26$

Question 7: $23 - [23 - \{23 - (23 - \overline{23 - 23})\}]$

Solution: Here, given expression contains vinculum. So, we first perform operations under it as follows:

$= 23 - [23 - \{23 - (23 - 0)\}]$

Removing common brackets, we get

$= 23 - [23 - \{23 - 23\}]$

Removing curly brackets, we get

$= 23 - [23 - 0]$

$= 23 - 23 = 0$

Question 8: $\mathbf{2550 - [510 - \{270 - (90 - \overline{80 + 70})\}]}$

Solution: Here, given expression contains vinculum. So, we first perform operations under it as follows:

$= 2550 - [510 - \{270 - (90 - 150)\}]$

Removing innermost brackets, we get

$= 2550 - [510 - \{270 - (-60)\}]$

$= 2550 - [510 - \{270 + 60\}]$

Removing curly brackets, we get

$= 2550 - [510 - 330]$

$= 2550 - 180$

$= 2370$

Question 9: $\mathbf{4 + \frac{1}{5}[\{-10 \times (25 - \overline{13 - 3})\} \div (-5)]}$

Solution: Here, given expression contains vinculum. So, we first perform operations under it as follows:

$= 4 + \frac{1}{5}[\{-10 \times (25 - 10)\} \div (-5)]$

Removing innermost brackets, we get

$= 4 + \frac{1}{5}[\{-10 \times 15\} \div (-5)]$

Removing curly brackets, we get

$= 4 + \frac{1}{5}[-150 \div (-5)]$

$= 4 + \frac{1}{5}\left[\frac{-150}{-5}\right]$

$= 4 + \frac{1}{5}(30)$

$= 4 + 6$

$= 10$

Question 10: $22 - \frac{1}{4}\{-5 - (-48) \div (-16)\}$

Solution: Performing division, we get

$= 22 - \frac{1}{4}\{-5 - (\frac{-48}{-16})\}$

$= 22 - \frac{1}{4}\{-5 - 3\}$

Removing curly brackets, we get

$= 22 - \frac{1}{4} \times (-8)$ (Performing multiplication)

$= 22 - (-2)$

$= 22 + 2$

$= 24$

Question 11: $63 - [(-3)\{-2 - \overline{8 - 3}\} \div [3\{5 + (-2)(-1)\}]$

Solution - Here, given expression contains vinculum. So, we first perform operations under it as follows:

$= 63 - [(-3)\{-2 - 5\} \div [3\{5 + (-2)(-1)\}]$

Removing innermost brackets, we get

$= 63 - [(-3) \times (-7) \div [3\{5 + 2\}]$

$= 63 - [(-3) \times (-7) \div [3 \times 7]$

$= 63 - [21 \div 21]$

$= 63 - 1$

$= 62$

Question 12: $[\mathbf{29} - (-\mathbf{2})\{\mathbf{6} - (\mathbf{7} - \mathbf{3})\}] \div [\mathbf{3} \times \{\mathbf{5} + (-\mathbf{3}) \times (-\mathbf{2})\}]$

Solution: Solving innermost brackets, we get

$= [29 - (-2)\{6 - 4\}] \div [3 \times \{5 + (-3) \times (-2)\}]$

Removing curly brackets, we get

$= [29 - (-2) \times 2] \div [3 \times \{5 + 6\}]$

$= [29 - (-4) \div [3 \times 11]$

$= [29 + 4] \div 33$

$= 33 \div 33$

$= 1$

Question 13 – Using brackets, write a mathematical expression for each of the following:

(i) Nine multiplied by the sum of two and five

Solution: $9 \times (2 + 5)$

(ii) Twelve divided by the sum of one and three

Solution: $12 \div (1 + 3)$

(iii) Twenty divided by the difference of seven and two.

Solution: $20 \div (7 - 2)$

(iv) Eight subtracted from the product of two and three.

Solution: $(2 \times 3) - 8$

(v) Forty divided by one more than the sum of nine and ten

Solution: $40 \div \{1 + (9 + 10)\}$

(vi) Two multiplied by one less than the difference of nineteen and six

Solution: $2 \times \{(19 - 6) - 1\}$

Objective Type Questions

Question 1: $(-1) \times (-1) \times (-1) \times \ldots . \boldsymbol{500\ times} =?$

Solution – Using the property of multiplication, $(-1) \times (-1) \times (-1) \times \ldots \times (-1) = 1$, if n is even

Here, n = 500, which is even

We have $(-1) \times (-1) \times (-1) \times \ldots 500 \text{ times} = 1$

Question 2: $(-1) + (-1) + (-1) + \cdots . \boldsymbol{500\ times} =?$

Solution: Using the property, $(-1) + (-1) + (-1) + \cdots + (-1) = n(-1)$

Here, n = 500

Thus, $(-1) + (-1) + (-1) + \cdots .500\ times = 500(-1) = -500$

Question 3: The additive inverse of -7 is?

Solution: The additive inverse of -7 is $-(-7) = 7$

Question 4: The modulus of an integer x is 9, then x =?

Solution: Given that $|x| = 9$

=> x = either 9 or -9

=> x = ± 9

Question 5: By how much does 5 exceed -4?

Solution: Let 5 exceeds -4 by 'x'

Thus we have, $5 = -4 + x$

=> $5 + 4 = x$

=> $x = 9$

Question 6: By how much less than -3 is -7?

Solution: Let 'x' be the required number

Thus we have, $(-3) - x = -7$

=> $-3 + 7 = x$

=> $x = 4$

Question 7: The sum of two integers is 24. If one of them is -19, then the other is

Solution: Let the other number be 'x'

Given that one integer is -19

Sum of two integers = 24

=> $x + (-19) = 24$

=> $x - 19 = 24$

=> $x = 24 + 19$

=> $x = 43$

Question 8: What must be subtracted from -6 to obtain -14?

Solution: Let the required number be 'x'

Then, we have $-6 - (x) = -14$

=> $-6 - x = -14$

=> $x = -6 + 14$

=> $x = 8$

Question 9: What should be divided by 6 to get -18?

Solution: Let the required number be 'x'

Then we have, $x \div 6 = -18$

=> $\frac{x}{6} = -18$

By cross multiplication, we get

=> $x = -18 \times 6 = -108$

Question 10: Which of the following is correct?

(a) -12 > -9

(b) -12 < -9

(c) (-12) + 9 > 0

(d) (-12) × 9 >0

Solution: $-12 > -9$ is not correct since -12 is less than -9

$-12 < -9$ is correct since -12 is less than -9

$-12 + 9 = -3 > 0$ is not correct as 0 is greater than -3

$-12 \times 9 = -108 > 0$ is now correct as 0 is greater than -108

Thus, option (b) is correct.

Question 11: The sum of two integers is -8. If one of the integers is 12, then the other is?

Solution: Let the other integer be 'x'

Given that, one integer = 12

Sum of two integers = -8

=> $x + 12 = -8$

=> $x = -8 - 12$

=> $x = -20$

Question 12: On subtracting -14 from -18, we get

Solution: we have $-18 - (-14)$

$= -18 + 14$

$= -4$

Question 13: $(-35) \times 2 + (-35) \times 8 =$?

Solution: Using the distributive property, $a \times (b + c) = a \times b + a \times c$

We have, $(-35) \times 2 + (-35) \times 8 = (-35)(2 + 8)$

$= (-35) \times 10 = -350$

Question 14: If $x \div 29 = 0$, then x =?

Solution: $\frac{x}{29} = 0$

=> $x = 0 \times 29 = 0$ (By cross multiplication)

Question 15: If $x = (-10) + (-10) + (-10) + \cdots$ 15 *times* and

y =$(-2) \times (-2) \times (-2) \times (-2) \times (-2)$, then x – y =?

Solution: x = $(-10) + (-10) + (-10) + \cdots 15\ times$

=> $x = (-10) \times 15 = -150$

And $y = -(2 \times 2 \times 2 \times 2 \times 2) = -32$

Thus, $x - y = -150 — 32 = -150 + 32$

=> $x - y = -118$

Question 16: If $\boldsymbol{a = (-1) \times (-1) \times (-1) \ldots . 100\ times}$ and $\boldsymbol{b = (-1) \times (-1) \times (-1) \ldots . 95\ times}$, then $\boldsymbol{a + b}$ =?

Solution: We know that,$(-1) \times (-1) \times (-1) \times \ldots \times (-1) = 1$, if n is even

= -1, if n is odd

Now, a = $(-1) \times (-1) \times (-1) \ldots .100\ times$

Here, n = 100 which is even

=> $a = 1$

$b = (-1) \times (-1) \times (-1) \ldots .95\ times$

Here, n = 95 which is odd

=> b = -1

Therefore, $a + b = 1 + (-1)$

=> $a + b = 0$

Question 17: $\boldsymbol{\big||3 - 12| - 4\big|}$ =?

Solution: Since$|-a| = a$, for any value of 'a'

$\big||-9| - 4\big|$

= $|9 - 4| = |5| = 5$

Question 18: If the difference of an integer 'a' and (-9) is 5, then a =?

Solution: Given that $a - (-9) = 5$

=> $a + 9 = 5$

=> $a = 5 - 9 = -4$

Question 19: The sum of two integers is 10. If one of them is negative, then other has to be?

Solution: Given that, sign of one integer is negative

And, sum of two integers = 10 which is positive

Thus, the sign of other integer must be positive because sum of two negatives will result in negative integer so one integer must be positive in order to get positive integer.

Question 20: If $x = (-1) \times (-1) \times (-1) \ldots 25\ times$, y =$(-3) \times (-3) \times (-3)$, then x y =?

Solution: We know that,$(-1) \times (-1) \times (-1) \times \ldots \times (-1) = -1$, if n is odd

Now, $x = (-1) \times (-1) \times (-1) \ldots 25\ times$

Here, n = 25 which is odd

Thus, x = -1

y =$(-3) \times (-3) \times (-3)$

$y = -(3 \times 3 \times 3)$

=> $y = -27$

Therefore, $xy = (-1) \times (-27) = 27$